ALLANGRAY

SHARP

2023 Wits Mathematics Competition
Qualifying Round
Upper Primary

Instructions

This exam consists of 20 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The mark allocation is as follows:

Questions 1-5 are each worth 3 points,
Questions 6-10 are each worth 4 points,
Questions 11-15 are each worth 5 points,
Questions 16-20 are each worth 6 points.
The total number of points available is 90.

The time limit on this exam is 75 minutes, calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed. If you are using the computer-friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

"If I were to awaken after having slept for a thousand years, my first question would be: Has the Riemann hypothesis been proven?". — David Hilbert

A. 3 point questions

1. What time did the clock show 3 hours and 50 minutes ago?

   (A) 06:20   (B) 06:22   (C) 10:40   (D) 14:00   (E) 14:02

   Solution: The current time on the clock is 10:12. Thus to get the time 3 hours and 50 minutes ago we first deal with the hours and then the minutes. So 10 o'clock subtract 3 hours is 7 o'clock. The time is then 07:12. Now looking at the minutes, we first subtract 12 from 50 $(50 - 12 = 38)$ to get us to 7 o'clock. We still have 38 minutes, since 1 hour = 60 minutes we take $60 - 38 = 22$. Thus the time is 06:22.
   So the answer is **B**

2. Lionel runs 60 meters in 7.2 seconds. If he was to maintain the same speed for 200 meters, what would be his time for 200 meters (in seconds)?

   (A) 22   (B) 8,3   (C) 48   (D) 24   (E) 1440

   Solution: We can work out how long it took Lionel to run 1 meter $(7.2 \div 60 = 0.12)$ and then multiply that by 200 to get how long it takes him to run 200 meters, $0.12 \times 200 = 24$. Another way to work this out is to work out how many 60 meters there are in 200 meters $(200 \div 60 = \frac{10}{3})$ and then multiply that by the time it takes him to run 60 meters, $\frac{10}{3} \times 7.2 = 24$.
   So the answer is **D**

3. Two cakes weigh the same as 32 ginger biscuits. Five cupcakes weigh the same as 24 ginger biscuits. How many cupcakes weigh the same as three cakes?

   (A) 10   (B) 24   (C) 20   (D) 48   (E) 16

   Solution: First we work out the weight of 1 cake in ginger biscuits. $32 \div 2 = 16$, so 3 cakes weigh $3 \times 16 = 48$ ginger biscuits. Now to get how many cupcakes weigh as much as 3 cakes we divide the number of ginger biscuits of 3 cakes by the number ginger biscuits for the cupcakes. $48 \div 24 = 2$, so 3 cakes weigh the same as $2 \times 5 = 10$ cupcakes.
   So the answer is **A**

4. Thabo rides his bicycle to school. His bicycle tyre has a circumference of 1,5m and the distance to school is 1,2km. How many revolutions does his bicycle tyre make to school and back?

(A) 1200   (B) 1600   (C) 1800   (D) 2000   (E) 2400

Solution: First we need to convert 1,2km to meters; $1,2 \times 1000 = 1200$m. Since we are looking for the number of revolutions on the way to school and back we double 1200m, which gives us 2400m. Now we divide 2400m by 1,5m to get how many revolutions his bicycle tyre did; $2400 \div 1,5 = 1600$.
So the answer is **B**

5. Jen loves making salads. If she makes 2 green salads it takes 40 minutes. If she makes one green salad and one potato salad, it takes her 52 minutes. How long does it take her to make four green salads and six potato salads?

(A) 1 h 32 m   (B) 2 h 32 m   (C) 3 h 32 m   (D) 4 h 32 m   (E) 5 h 32 m

Solution: Since it takes 52 minutes to make one green salad and one potato salad, then it takes $52 + 52 = 104$ minutes to make 2 green salads and 2 potato salads. But we know it takes 40 minutes to make 2 green salads, so then taking away the 40 minutes it takes to make the 2 green salads from the 104 minutes gives us the time it takes to make 2 potato salads, $104 - 40 = 64$. So, it takes 1 hour and 4 minutes to make 2 poato salads, to make 6 potato salads we multiply this time by 3, which gives 3 hours and 12 minutes. Now to make the 4 green salads we multiply the 40 minutes to make 2 green salads by 2, which gives 80 minutes. So it takes 1 hour and 20 minutes to make 4 green salads. So to make 4 green salads and 6 potato salads is 3 hours 12 minutes plus 1 hour 20 minutes, so in total it takes 4 hours 32 minutes.
So the answer is **D**

B. 4 point questions

6. How many rectangles of different perimeters but with a common area of 18 are there? (Sides of these rectangles are all whole numbers. For purposes of this question a three by four and a four by three rectangle are considered the same).

   (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

   Solution: The area of a rectangle is length $\times$ width, so we just need to look at the factors of 18. The factors of 18 are; 1 and 18, 2 and 9, 3 and 6. So there are 3 pairs that when multiplied together give us 18.
   So the answer is **A**

7. A cake weighing 1200g is cut into five pieces. The largest slice weighs half as much as the other four together. Find the weight of the largest slice.

   (A) 200g   (B) 300g   (C) 400g   (D) 600g   (E) 800g

   Solution: Since the largest slice weighs half as much as the other 4 together it weighs as much as 2 of the other slices. So we count the largest slice as 2 and then divide 1200g by 6 to get the weight of the normal slices. $1200 \div 6 = 200$, so a normal slice weighs 200g and since the largest slice is twice the weight of a normal slice it weighs, $2 \times 200 = 400$g.
   So the answer is **C**

8. Five students share R12 000 in prize money. Buhle receives $\frac{1}{6}$ of the money, Shaan receives $\frac{1}{5}$ of the money, Julian receives $\frac{1}{4}$ of the money, Thabo receives $\frac{1}{3}$ of the money. How much money does Lexi receive if she gets the remainder of the money?

   (A) 11400   (B) 600   (C) 6000   (D) 800   (E) 400

   Solution: First we make each fraction have the same denominator. So, $\frac{1\times10}{6\times10} = \frac{10}{60}$, $\frac{1\times12}{5\times12} = \frac{12}{60}$, $\frac{1\times15}{4\times15} = \frac{15}{60}$ and $\frac{1\times20}{3\times20} = \frac{20}{60}$. So the total amount money taken by the four is $\frac{10+12+15+20}{60} = \frac{57}{60}$. Thus the fraction of money that Lexi receives is $\frac{3}{60} = \frac{1}{20}$. So $\frac{1}{20}$ of 12 000 $= \frac{1}{20} \times 12\ 000 = 600$
   So the answer is **B**

9. A hammer and a nail cost R31. If a hammer costs R30 more than a nail, then what is the cost of the nail?

   (A) R0, 50   (B) R1, 00   (C) R2, 00   (D) R30, 00   (E) R31, 00

   Solution: Lets break down what the question is telling us;

   $$\text{Hammer} + \text{Nail} = \text{R31}$$
   $$\text{Hammer} = \text{Nail} + \text{R30}$$

Now in the place of Hammer in the first equation let us put what we find the value of the Hammer to be in the second equation. So we get;

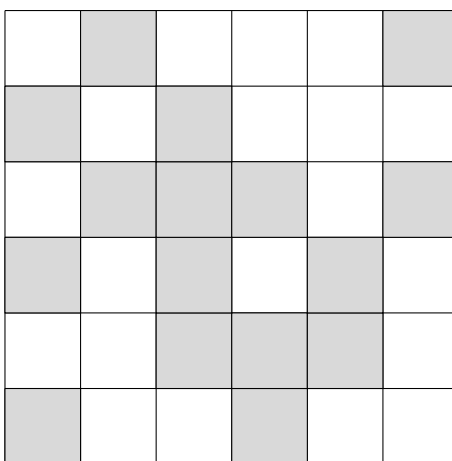$$\text{Nail} + \text{R30} + \text{Nail} = \text{R31}$$

So we can remove R30 from both sides to get;

$$\text{Nail} + \text{Nail} = \text{R1}$$

So the price of 2 nails is R1 making the price of 1 nail, R0, 50.
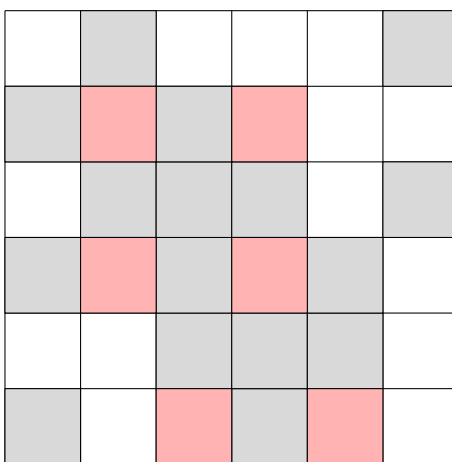So the answer is **A**

10. Temba and Thabiso play a game where the aim is to colour four squares grey to form a larger grey square. It is Temba's turn to colour a square in the grid below. How many options does he have to win the game?

(A) 2   (B) 4   (C) 6   (D) 8   (E) 10

Solution: The 6 squares Temba could colour in on his next turn to win the game are indicated in red on the diagram below;

So the answer is **C**

C. 5 point questions

11. Nine students take a mathematics test. The mean (average) score is 65. The mean of the top five students' scores is 87 and the mean of the bottom five students' scores is 44. Find the median student's score.

   (A) 50　　　　　(B) 65　　　　　(C) 70　　　　　(D) 75　　　　　(E) 80

    Solution: If the mean score of 9 students is 65 then the sum of all the students scores is 9×65. Similarly, the sum of the top 5 students is 5×87 and the sum of the bottom 5 students is 5×44. The key point is that the score the median student is in both the sum of the top 5 students and the bottom 5 students. So adding the sum of the top and bottom students is the sum of all the students scores plus the score of the median student again. Thus if we subtract the sum of all the students scores from the sum of the top and bottom students it will give us the score of the median student. So the median student's score is $(5 \times 87) + (5 \times 44) - (9 \times 65) = 435 + 220 - 585 = 655 - 585 = 70$. So the answer is **C**

12. Thabo has 29 green gems and 1 yellow gem. How many of the green gems will Thabo have to throw away to make eighty percent of the gems green?

    (A) 5　　　　　(B) 10　　　　　(C) 15　　　　　(D) 25　　　　　(E) 28

    Solution: If Thabo is to have 80% green gems that means he must have 20% yellow gems. Thus 20% is 1 gem and $80 \div 20 = 4$ so we need to have only 4 green gems. So we need to throw away $29 - 4 = 25$ green gems.
    So the answer is **D**

13. In the game called Geogons the product of the 2 numbers in the 2 circles gives the number in the square between them. What is the value $A + B + C$?
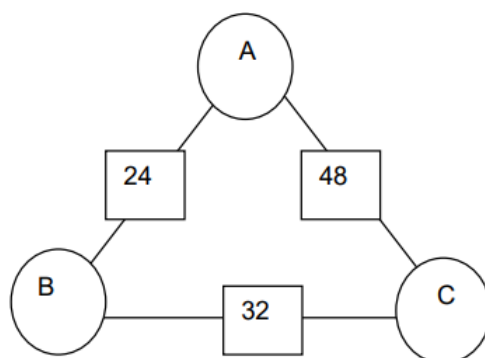
    (A) 4　　　　　(B) 12　　　　　(C) 18　　　　　(D) 32　　　　　(E) 54

Solution: We first get the factors of 24, 32 and 48;

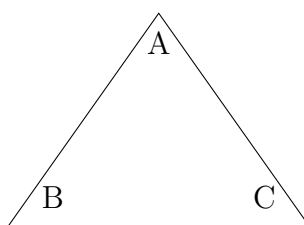|	
Factors of 24 :	1, 2, 3, 4, 6, 8, 12, 24
Factors of 32 :	1, 2, 4, 8, 16, 32
Factors of 48 :	1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Thus B can only be 2, 4 or 8. Lets make $B = 2$, then A must be 12. Since A is 12 then C must be 4, but $4 \times 2 = 8$ not 32. So B is not 2.
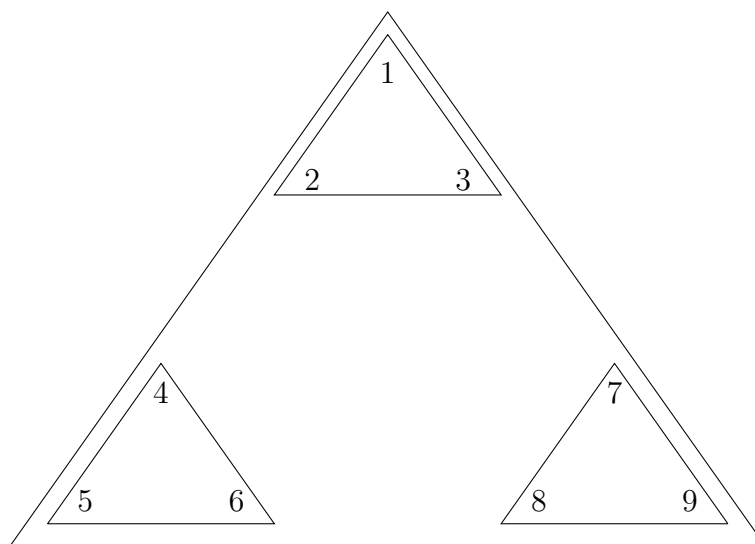
Now make $B = 4$, then A must be 6. Since A is 6 then C must be 8 and $8 \times 6 = 32$. So we know $A = 6$, $B = 4$ and $C = 8$. Then $A + B + C = 6 + 4 + 8 = 18$

So the answer is **C**

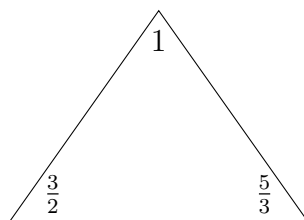14. The value of the figure below is given by $\frac{A+B}{C}$.

Find the value of:

(A) $\frac{2}{3}$ (B) 1   (C) $\frac{3}{2}$ (D) 2   (E) $\frac{21}{8}$

Solution: For the top triangle; $\frac{1+2}{3} = \frac{3}{3} = 1$. For the bottom left triangle; $\frac{4+5}{6} = \frac{9}{6} = \frac{3}{2}$. For the bottom right triangle; $\frac{7+8}{9} = \frac{15}{9} = \frac{5}{3}$. Now the big triangle looks as follows;

So now we get; $\frac{1+\frac{3}{2}}{\frac{5}{3}} = (\frac{2}{2} + \frac{3}{2}) \div \frac{5}{3} = \frac{5}{2} \times \frac{3}{5} = \frac{3}{2}$.
So the answer is **C**

15. The four-digit number $3XX1$ is divisible by 9. What digit does X represent?

    (A) 0   (B) 2   (C) 5   (D) 7   (E) 9

    Solution: We can use a process of elimination here.
    If $X = 0$ then our four-digit number is 3001 and $3001 \div 9 = 333$ remainder 4.
    If $X = 2$ then our four-digit number is 3221 and $3221 \div 9 = 357$ remainder 8.
    If $X = 5$ then our four-digit number is 3551 and $3551 \div 9 = 394$ remainder 5.
    If $X = 7$ then our four-digit number is 3771 and $3771 \div 9 = 419$. So, $X = 7$.
    So the answer is **D**

D. 6 point questions

16. A total of 20 students can choose to play soccer, basketball or badminton. Each student must play at least two of the games. There are 14 who play soccer, 15 who play basketball, and 16 who play badminton. How many students play all three games?

    (A) 4   (B) 5   (C) 6   (D) 8   (E) 10

    Solution: Lets add the number of people who play soccer and basketball together, $14 + 15 = 29$. But there are only 20 people so we must have counted $29 - 20 = 9$ people twice. That means 9 people play both soccer and basketball. Now lets add the number of people who play badminton and the number of people who play both soccer and basketball, $16 + 9 = 25$. Again there are only 20 people so we must have counted $25 - 20 = 5$ people twice. That means 5 people play all three sports.
    So the answer is **B**

17. The sum of the digits of a 3 digit number abc (a; b and c digits) is as follows:

    $$a + b = 9, \quad b + c = 10, \quad a + c = 7$$

    What is the 3-digit number?

    (A) 907   (B) 123   (C) 463   (D) 364   (E) 719

    Solution: If we add $a+b$ and $a+c$ then we end up with $b+c$ plus 2 a's. Since $a+b = 9$ and $a+c = 7$ then the sum of the two is equal to 16, but we also know $b+c = 10$ so if we subtract 10 from 16 we get the value of the 2 a's. So the 2 a's are equal to $16 - 10 = 6$ and then $a = 3$. Looking at our options D is the only one with 3 in the hundreds place. We can also work out the values of b and c by subtracting the value of a from each sum with a. So $b = 9 - 3 = 6$ and $c = 7 - 3 = 4$. The 3-digit number is then 364.
    So the answer is **D**

18. Compute $2023^2 - 2023 \times 2022$.

    (A) 2022   (B) 2023   (C) 4   (D) 4090506   (E) 4092529

    Solution: $2023^2 = 2023 \times 2023$, so $2023^2 - 2023 \times 2022 = 2023 \times 2023 - 2023 \times 2022$. Remember that 2023×2023 means we add 2023 lots of 2023 and 2023×2022 means we add 2022 lots of 2023. Since $2022 = 2023 - 1$ in 2023×2022 we add one less lot of 2023 than in 2023×2023. Thus $2023 \times 2023 - 2023 \times 2022 = 2023$.
    So the answer is **B**

19. Mdu enjoys solving mathematics problems! Every year since he can remember he's solved as many problems as he solved in the previous two years altogether. In 2022 he solved 159 problems, in 2021 he solved 103. How many did he solve in 2019?

    (A) 9   (B) 47   (C) 56   (D) 76   (E) 103

Solution: If Mdu solved 159 problems in 2022 and 103 problems in 2021 then in 2020 he solved $159 - 103 = 56$ problems. Similarly he then solved $103 - 56 = 47$ problems in 2019.

So the answer is **B**

20. If a,b and c are distinct natural numbers (positive integers) and $\dfrac{6^a 15^b}{9^b 10^c}$ is an integer. Find the order of a, b and c.

    (A) $a < b < c$ (B) $b < a < c$ (C) $b < c < a$ (D) $c < a < b$ (E) $c < b < a$

Solution: If $\dfrac{6^a 15^b}{9^b 10^c}$ is a integer then the denominator must 'disappear'. Let us break $6, 15, 9$ and 10 into their prime factors. This gives us; $6 = 2 \times 3$, $15 = 3 \times 5$, $9 = 3 \times 3$ and $10 = 2 \times 5$.

Let us look at the 5, 5 is only in 15 and 10 so to 'get rid' of the 5 in the denominator there must be more 5's in the numerator. This means that the b of the 15^b must be larger than the c of the 10^c. So we have that $c < b$.

Now let us look at the 3. 3 is in both the 6 and 15 in the numerator but only in the 9 in the denominator. However the b in 9^b and 15^b are the same so we won't see a difference the 3's there. So there must be more 3's in the numerator and we look to the 6^a to do that. This means that the a of the 6^a must be larger than the b of the 9^b. So we have that $b < a$.

Putting these together we get that $c < b < a$.

So the answer is **E**