ALLANGRAY

SHARP

2023 Wits Mathematics Competition
Qualifying Round
Middle Primary

Instructions

This exam consists of 20 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The mark allocation is as follows:

Questions 1-5 are each worth 3 points,
Questions 6-10 are each worth 4 points,
Questions 11-15 are each worth 5 points,
Questions 16-20 are each worth 6 points.
The total number of points available is 90.

The time limit on this exam is 75 minutes, calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed. If you are using the computer-friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

"If I were to awaken after having slept for a thousand years, my first question would be: Has the Riemann hypothesis been proven?". — David Hilbert

A. 3 point questions

1. Compute $20 + 23$.

   (A) 3 (B) 32 (C) 43 (D) 53 (E) 2023

   Solution: $20 + 23 = 20 + 20 + 3 = 40 + 3 = 43$
   So the answer is **E**

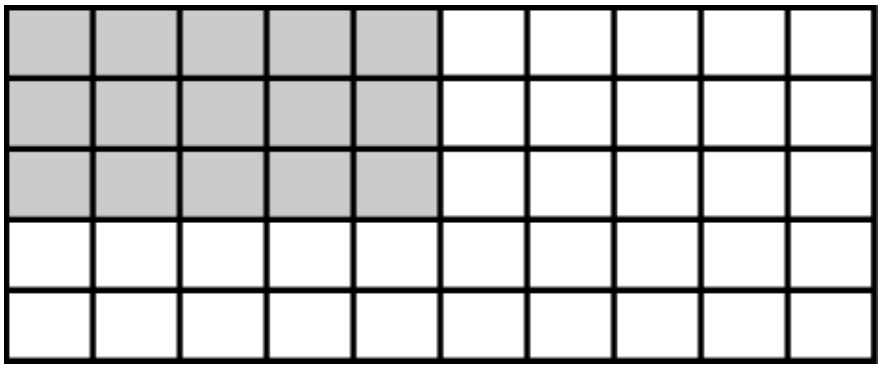
2. Which symbols should be placed inside the boxes?

   $$5\square 3\square 6 = 21$$

   (A) $\times$ and $-$ (B) $-$ and $\times$ (C) $\times$ and $+$ (D) $+$ and $+$ (E) $\div$ and $+$

   Solution: Looking at 5 and 3; if we have $5 + 3$ then $5 + 3 = 8$. Since $8 < 21$ we only need to look at $\times$ and $+$. $8 \times 6 = 48 \neq 21$ and $8 + 6 = 14 \neq 21$.
   If we have $5 - 3$ then $5 - 3 = 2$. Since $2 < 21$ we only need to look at $\times$ and $+$. $2 \times 6 = 12 \neq 21$ and $2 + 6 = 8 \neq 21$. If we have 5×3 then $5 \times 3 = 15$. Since $15 < 21$ we only need to look at $\times$ and $+$. $15 + 6 = 21$.
   So the answer is **C**

3. How many more squares should be shaded so that the number of shaded and unshaded squares is the same?



   (A) 10 (B) 15 (C) 25 (D) 30 (E) 50

   Solution: For there to be the same number of shaded and unshaded squares we need to half the total number of squares and then shaded one half. There are 10 columns of 5 squares so there are $10 \times 5 = 50$ squares in total. Half of 50 is 25 so we need 25 shaded squares. We have 5 columns of 3 shaded squares so we have $5 \times 3 = 15$ shaded squares. So to have 25 shaded squares we need to shaded $25 - 15 = 10$ more squares.
   So the answer is **A**

4. Sweet bags at Wits always contain exactly 30 sweets. Each sweet is either red or blue (never both). Sam buys two bags of sweets. The first bag has 17 blue sweets and the second bag has 16 red sweets. How many red sweets are there in the two bags together?

   (A) 13 (B) 16 (C) 29 (D) 30 (E) 31

   Solution: Since the first bag has 17 blue sweets and each sweet is either red or blue it means that the rest of the sweets in the bag are red. So there are $30 - 17 = 13$ red sweets in the first bag. To get the amount of red sweets in the two bags we add the number of red sweets in the first bag and the number of red sweets in the second bag. So we have $13 + 16 = 29$ red sweets.
   So the answer is **C**

5. Rowan initially has R80. He spends half of his money on ice-cream. He then gives a quarter of his remaining money to his friend. How much money does he have at the end?

   (A) R10 (B) R20 (C) R30 (D) R40 (E) R50

   Solution: Half of 80 is 40. Now we need to take a quarter $(\frac{1}{4})$ of 40. $40 \div 4 = 10$ so we need to subtract 10 from 40 giving us, $40 - 10 = 30$.
   So the answer is **C**

B. 4 point questions

6. A soccer stadium has 60 000 seats. On the day of the concert 59 300 seats were sold, 550 complimentary (free) tickets were given and there were 420 empty seats. How many people did not show up?

   (A) 420 (B) 130 (C) 270 (D) 680 (E) 1670

   Solution: We first need to know how many seats were taken up, which is $59\,300+550 = 59\,850$. This means that $60\,000-59\,850 = 150$ seats were going to be empty because no one had a ticket for them. Now if we remove these 150 from the number of empty seats it will tell us how many people had tickets and didn't show up. So, $420 - 150 = 270$.
   So the answer is **C**

7. Cindy has a certain amount of money. After spending $\frac{1}{4}$ of the money she has R60 left. How much did she start off with?

   (A) R40 (B) R60 (C) R80 (D) R100 (E) R120

   Solution: Since after Cindy spent $\frac{1}{4}$ of the money she has R60, that means R60 is $\frac{3}{4}$ of the money. So we divide 60 by 3 to get what $\frac{1}{4}$ would be, $60 \div 3 = 20$. So the money she started with is $60 + 20 = 80$. So the answer is **C**

8. What time did the clock show 3 hours and 50 minutes ago?

   (A) 06:20 (B) 06:22 (C) 10:40 (D) 14:00 (E) 14:02

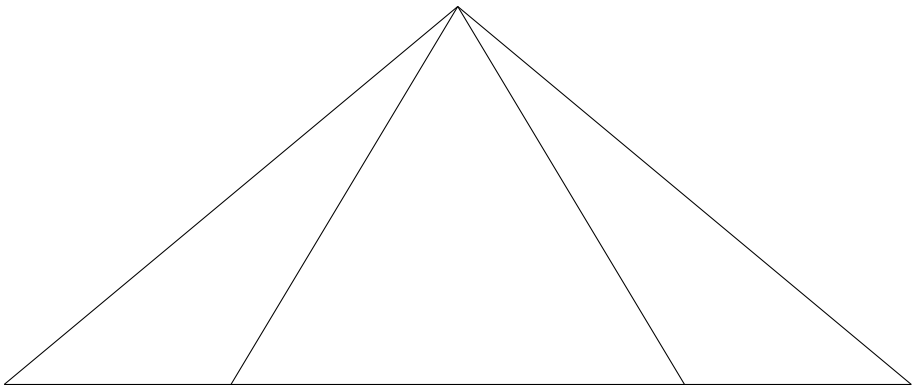
   Solution: The current time on the clock is 10:12. Thus to get the time 3 hours and 50 minutes ago we first deal with the hours and then the minutes. So 10 o'clock subtract 3 hours is 7 o'clock. The time is then 07:12. Now looking at the minutes, we first subtract 12 from 50 ($50 - 12 = 38$) to get us to 7 o'clock. We still have 38 minutes, since 1 hour = 60 minutes we take $60 - 38 = 22$. Thus the time is 06:22.
   So the answer is **B**

9. Lionel runs 60 meters in 7.2 seconds. If he was to maintain the same speed for 200 meters, what would be his time for 200 meters (in seconds)?

(A) 22 (B) 8,3 (C) 48 (D) 24 (E) 1440

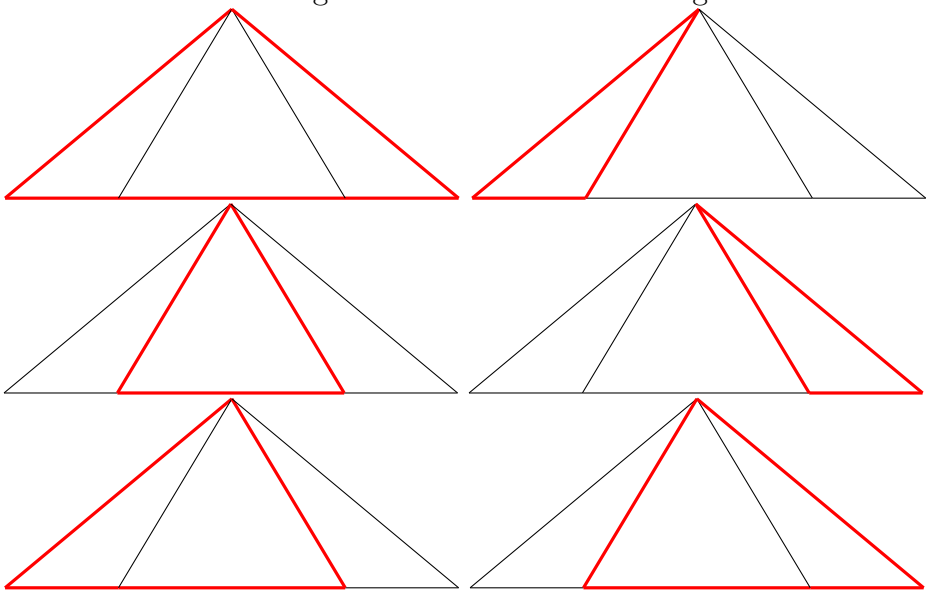
Solution: We can work out how long it took Lionel to run 1 meter ($7.2 \div 60 = 0.12$) and then multiply that by 200 to get how long it takes him to run 200 meters, $0.12 \times 200 = 24$. Another way to work this out is to work out how many 60 meters there are in 200 meters ($200 \div 60 = \frac{10}{3}$) and then multiply that by the time it takes him to run 60 meters, $\frac{10}{3} \times 7.2 = 24$.
So the answer is **D**

10. How many triangles of all sizes are there in the figure below? Some triangles may contain others.



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Solution: The 6 triangles are indicated in the diagrams below.



So the answer is **D**

C. 5 point questions

11. 36 tiles (all of the same size) are used to cover a square floor. How many tiles are needed to tile a floor twice as long and twice as wide?

    (A) 72 (B) 64 (C) 38 (D) 288 (E) 144

    Solution: Since the area of a square is length $\times$ width where the length and width are the same, we need to look at the factors of 36. The factors of 36 are; 1 and 36, 2 and 18, 3 and 12, 4 and 9, 6. So the length must be 6. Now to get the area/number of tiles we double 6 and then multiply that by itself, so $(2 \times 6) \times (2 \times 6) = (12) \times (12) = 144$ tiles.
    So the answer is **E**

12. I sell pies 6 days a week. On a Saturday I sell 3 times as much as on any other day (I sell the same amount the other five days). If I sell 480 pies a week, how many do I sell on a Saturday?

    (A) 240 (B) 160 (C) 210 (D) 480 (E) 180

    Solution: Let us count Saturday as 3 days so that we can work out how many pies we sell each day. So we divide 480 by 8 $(5+3=8)$ and get $480 \div 8 = 60$. So we sell 60 pies on a normal day. Thus we sell $3 \times 60 = 180$ pies on Saturday.
    So the answer is **E**

13. Alex's age is 3 times the age of John. In 10 years Alex's age will be 2 times John's age. How old is John now (in years)?

    (A) 8 (B) 10 (C) 12 (D) 20 (E) 25

    Solution: Let us break down what the question is telling us.

    $$\text{Alex's age} = 3 \times (\text{John's age})$$
    $$(\text{Alex's age}) + 10 = 2 \times ((\text{John's age}) + 10)$$

    So Alex's age is a multiple of 3, which are $3, 6, 9, 12, 18, 21, 24, 27, 30, 33, 36, \ldots$ and when we add 10 to it is a multiple of 2, in other words it's even. So now our list of multiples of 3 that fit becomes; $6, 12, 18, 24, 30, 36, \ldots$. Now lets check if any fit the second equation;

Alex's Age	John's Age	(Alex's age) + 10	$2 \times ((\text{John's age}) + 10)$
6	2	16	$2 \times (2+10) = 2 \times 12 = 24$
12	4	22	$2 \times (4+10) = 2 \times 14 = 28$
18	6	28	$2 \times (6+10) = 2 \times 16 = 32$
24	8	34	$2 \times (8+10) = 2 \times 18 = 36$
30	10	40	$2 \times (10+10) = 2 \times 20 = 40$
36	12	46	$2 \times (12+10) = 2 \times 22 = 44$

    Looking at the table above we see that when John's age is 10 then we get that (Alex's age)$+10 = 2 \times ((\text{John's age}) + 10)$.
    So the answer is **B**

14. In a reptile park there are snakes and lizards. If there are 84 legs and 84 heads in total, how many snakes are there in the reptile park? Lizards have four legs and one head. Snakes have zero legs and one head.

(A) 105 (B) 84 (C) 63 (D) 42 (E) 21

Solution: Since snakes have no legs to get how many lizards we just divide the number of legs by 4, $84 \div 4 = 21$. So we have 21 lizards and to get the number of snakes we subtract the number of lizards from the number of heads, $84 - 21 = 63$.
So the answer is **C**

15. The sequence below is generated by writing each letter of the alphabet one more time than the previous letter:
A,A,B,B,B,C,C,C,C,D,D,D,D,D,E,E,E,E,E,E,F,....
What letter is in position 75?

(A) J (B) K (C) L (D) M (E) N

Solution: We are going to add consecutive numbers till we reach 75 or get a number above 75. The letter will be the letter that goes with the new number added;

A: 2
B: $2+3=5$
C: $2+3+4=9$
D: $2+3+4+5=14$
E: $2+3+4+5+6=20$
F: $2+3+4+5+6+7=27$
G: $2+3+4+5+6+7+8=35$
H: $2+3+4+5+6+7+8+9=44$
I: $2+3+4+5+6+7+8+9+10=55$
J: $2+3+4+5+6+7+8+9+10+11=65$
K: $2+3+4+5+6+7+8+9+10+11+12=77$

So K is in position 75.
So the answer is **B**

D. 6 point questions

16. Nine students take a mathematics test. The mean (average) score is 65. The mean of the top five students' scores is 87 and the mean of the bottom five students' scores is 44. Find the median student's score (that is the score of the person in the middle of the group).

    (A) 50 (B) 65 (C) 70 (D) 75 (E) 80

    Solution: If the mean score of 9 students is 65 then the sum of all the students scores is 9×65. Similarly, the sum of the top 5 students is 5×87 and the sum of the bottom 5 students is 5×44. The key point is that the score the median student is in both the sum of the top 5 students and the bottom 5 students. So adding the sum of the top and bottom students is the sum of all the students scores plus the score of the median student again. Thus if we subtract the sum of all the students scores from the sum of the top and bottom students it will give us the score of the median student. So the median student's score is $(5 \times 87) + (5 \times 44) - (9 \times 65) = 435 + 220 - 585 = 655 - 585 = 70$.
    So the answer is **C**

17. The time on a 12-hour digital clock is 8:05. How much time needs to pass before all the digits on the clock are the same?

    (A) 2 h 16 min (B) 1 h 24 min (C) 7 h 6 min (D) 3 h 6 min (E) 8 h 6 min

    Solution: On a 12-hour digital clock the times where all the digits are the same are; 1:11, 2:22, 3:33, 4:44, 5:55, 11:11, 12:12. The time that happens first after 8:05 is 11:11. Lets deal with the hours first, $11-8 = 3$ hours. Now for the minutes, $11-5 = 6$ minutes. Thus 3 hours 6 minutes needs to pass.
    So the answer is **D**

18. Four children write a test. The average mark for the four of them was 80%. A fifth child joins the group with a test mark of 40%. What is the new average mark for the five of them?

    (A) 40 (B) 80 (C) 70 (D) 72 (E) 60

    Solution: Since the average of the 4 original children is 80% for our calculation of the new average with the fifth child's mark added we make each of the original children's mark 80%. So the new average is;

    $$\frac{4(80) + 40}{5} = \frac{360}{5} = 72$$

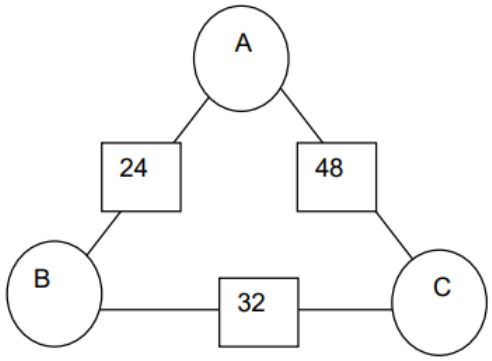
    So the answer is **D**

19. What is the difference between the $50th$ number and the $100th$ number in the list $6, 12, 18, 24, \ldots$? The difference between each pair of numbers is the name.

(A) 56 (B) 206 (C) 300 (D) 306 (E) 606

Solution: Looking at the list we can see that it is just that multiples of 6 (the 6 times table). So the $50th$ number is just $6 \times 50 = 300$ and the $100th$ number is $6 \times 100 = 600$. So the difference is $600 - 300 = 300$.
So the answer is **C**

20. In the game called Geogons the product of the two numbers in two circles gives the number in the square between them. What is the value $A + B + C$?



(A) 4 (B) 12 (C) 18 (D) 32 (E) 54

Solution: We first get the factors of 24, 32 and 48;

Factors of 24 :	1, 2, 3, 4, 6, 8, 12, 24
Factors of 32 :	1, 2, 4, 8, 16, 32
Factors of 48 :	1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Thus B can only be 2, 4 or 8. Lets make $B = 2$, then A must be 12. Since A is 12 then C must be 4, but $4 \times 2 = 8$ not 32. So B is not 2.
Now make $B = 4$, then A must be 6. Since A is 6 then C must be 8 and $8 \times 6 = 32$. So we know $A = 6$, $B = 4$ and $C = 8$. Then $A + B + C = 6 + 4 + 8 = 18$
So the answer is **C**