SHARP

2022 Wits Mathematics Competition
Qualifying Round
Upper Primary

Instructions

This exam consists of 20 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The mark allocation is as follows:

Questions 1-5 are each worth 3 points,
Questions 6-10 are each worth 4 points,
Questions 11-15 are each worth 5 points,
Questions 16-20 are each worth 6 points.
The total number of points available is 90.

The time limit on this exam is 75 minutes, calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed. If you are using the computer-friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profoundity, he is talking nonsense" — Alfred North Whitehead

A. 3 point questions

1. Compute 20×22.

   A) 380 B) 400 C) 420 D) 440 E) 460

   Solution: D $20 \times 22 = 440$

2. What percentage of the squares in the diagram below are shaded?

   A) 20 B) 40 C) 60 D) 80 E) 100

   Solution: D. There are 8 shaded squares and 2 unshaded. $\frac{8}{8+2} = 0.8 = 80\%$

3. Compute $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$.

   A) 1 B) 5 C) 10 D) 25 E) 55

   Solution: B. This can be done by adding the terms but it's easier to see (and less error prone) by grouping the terms as follows: $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1 = (10 - 9) + (8 - 7) + (6 - 5) + (4 - 3) + (2 - 1) = 1 + 1 + 1 + 1 + 1 = 5$

4. How many rectangles are in the following diagram? (*Squares are also rectangles.*)

   A) 9 B) 12 C) 15 D) 18 E) 21

   Solution: D. Six 1×1 rectangles, four 2×1 rectangles, two 3×1 rectangles, three 1×2 rectangles, two 2×2 rectangles and one 3×2 rectangles. This adds up to 18.

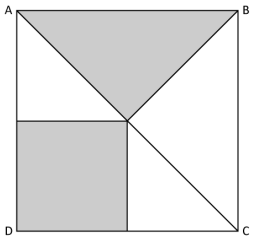
5. In the image below, which square can be removed to increase the total perimeter of the shape? Perimeter means the distance around the shape.

A) Square A B) Square E C) Square F D) Square J E) Square K

Solution: C. Removing square F increases the perimeter by 2. Removing squares E or L would not change the perimeter and removing squares A or J would decrease the perimeter by 2.

B. 4 point questions

6. The area of the square $ABCD$ is 40cm^2. What is the area of the shaded region in cm^2?



A) 10 B) 15 C) 20 D) 25 E) 30

Solution: C. The shaded area is half of the total area and therefore 20cm^2. To see this notice that the shaded square in the bottom left is a quarter of the total area and the shaded triangle is also a quarter.

7. The surface area of a cube is 54cm^2. What is the volume of the cube in cm^3?

A) 27 B) 64 C) 100 D) 125 E) 1000

Solution: A. As cubes have six faces the area of an individual face is 9cm^2. Making the faces 3×3 squares. Thus the cube is a $3 \times 3 \times 3$ cube, and has volume 27cm^3

8. Sihle opens her favourite mathematics puzzle book and notes that the product of the page numbers facing her is 380. Find the sum of these two page numbers.

A) 35 B) 36 C) 37 D) 38 E) 39

Solution: E. The only two consecutive numbers we can multiple together to get 380 are 19 and 20, which sum to 39.

9. What is the minimum number of digits that must be removed from the number 12323314, so that the resulting number is the same when read from either left to right or right to left?

A) 1 B) 2 C) 3 D) 4 E) 5

Solution: C. First notice that the 4 has to be removed (or else everythign else removed with is inefficent). This leaves us with 1232331 and it's easy to check that removing a single extra digit here isn't enough and that two is enough (the last two threes or both twos for example).

10. The Olympic committee printed numbered bibs for each of the athletes competing in the ski jump event, starting from the number 1. If 234 digits were printed overall, how many athletes competed?

A) 90 B) 114 C) 123 D) 172 E) 234

Solution: B. The first 9 athletes use a single digit per bib. The next 90 use two digits a bib. Which means after printing 99 bibs we'll have use 189 digits and have 45 'left'. As the next 900 will use three digits a bib we can see that our remaining digits are enough for another 15 bibs. A total of $99 + 15 = 114$.

C. 5 point questions

11. Which of the following fractions is closest to $\frac{1}{2}$?

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{2}{5}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$

Solution: C. A and B are $\frac{1}{6} = 0.166..$ away from $\frac{1}{2}$ D and E are respectively 0.3 and $\frac{1}{3} = 0.333...$ away and C is a mere 0.1 away.

12. A box containing four wits maths textbooks weighs 10 kg, the same box with six wits maths textbooks weighs 13 kg. How much does an empty box weigh (in kg) if all the books weigh the same?

A) 2 B) 3 C) 4 D) 5 E) 6

Solution: C. Taking differences it's easy to see that two books weigh three kilograms. Therefore removing four books from the first box would decrease it's weight by six kilograms. Making an empty Box weigh four kilos.

13. The product of two positive integers is equal to twice their sum. The same product is also equal to six times the difference between the two integers. What is the sum of the integers?

A) 3 B) 6 C) 9 D) 15 E) 18

Solution: C. As the numbers are small this can be solved by trial and error. A more systematic approach is to use algebra. Call the numbers a and b and choose $a < b$. Then $ab = 6b - 6a = 2a + 2b$. $6b - 6a = 2a + 2b$ implies $b = 2a$. So subbing into $ab = 6b - 6a$ gives $2a^2 = 6a$ and $a^2 = 3a$. Which gives us $a = 3$ and $b = 6$.

14. Callan has 10 blue marbles, 6 green marbles and 5 red marbles in a bag. He draws one marble at a time and puts it aside. How many times should he draw to ensure that he has at least one marble of each colour?

A) 4 B) 12 C) 15 D) 16 E) 17

Solution: E. If he's very unlucky his first 16 draws could be all the blue and green marbles. So 17 draws are needed to ensure at least one of each colour.

15. A palindrome is a number that is the same when read backwards e.g 565 or 45754. What is the difference between the largest four digit palindrome and the smallest four digit palindrome? Initial zeros are not allowed so for example 0110 is not considered a four digit palindrome because it's not a four digit number.

A) 8668 B) 8778 C) 8888 D) 8998 E) 9000

Solution: D. The largest four digit palindrome is 9999 while the smallest is 1001. The difference between these is $9999 - 1001 = 8998$.

D. 6 point questions

16. How many 3 digit numbers (whole numbers between 100 and 999) are multiples of 6, 10 **and** 15?

A) 18 B) 20 C) 30 D) 33 E) 60

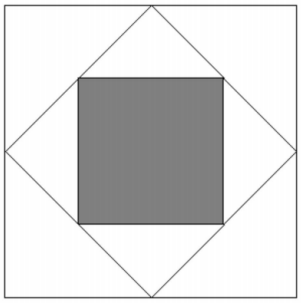
Solution: C. A number is a multiple of all three of these if and only if it's a multiple of 30 (the lcm of the three numbers). Which means the set we're looking at is 120, 150, 180, ...,960,990. 990 is the 33^{rd} multiple of 30 but we must then take away 30, 60 and 90 leaving 30 elements in this set.

17. 30 students wrote a mathematics exam out of 100 marks. The average score was 50. The average score of those who passed was 60 and the average score of those who failed was 45. How many students passed?

A) 10 B) 12 C) 14 D) 16 E) 18

Solution: A. The total number of marks scored was $30 \times 50 = 1500$. If x students passed then $1500 = 60x + 45(30 - x)$ which solves to $x = 10$. This could also be solved via trial and error if the student was not familiar with algebra (trying different values for the number of passers).

18. The diagram below shows a shaded square in a square inside another square. What fraction of the diagram above is shaded?



A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$
E) Impossible to determine

Solution: C. The rotated square is half the area of the outside square and the shaded square is in turn half the area of the rotated square.

19. When 1 is subtracted from 5 times a number and the result is divided by 3 more than twice the original number we get $\frac{4}{5}$. Find the number.

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) 5

Solution: B. Again this could be solved by trial and error, an approach that is doubly convenient as this round was multiple choice. Again an algebraic approach is more systematic. Let the number be x. Then we are given $\frac{5x-1}{2x+3} = \frac{4}{5}$ which can be rewritten as $25x - 5 = 8x + 12$ and simplified to $17x = 17$ and finally to $x = 1$.

20. A stand sells movie popcorn in only 2 sizes. Their prices are R4 and R7 per serving. What is the greatest popcorn sales value, in Rands, that is NOT POSSIBLE?

A) 15 B) 17 C) 23 D) 43 E) None

Solution: B. To see if an Rn purchase is possible we'll consider the remainder when n is divided by four. First notice that all multiples of 4 can be done, using only the 4. Now if we buy a single serving of size seven and some of size four, we see that 7,11,15,... can be done (but R3 cannot). Similarly if we buy two R7 servings We can get the numbers 14,18,22 and so on (but not 2, 6 or 10). Finally if we buy three servings of size 7 we can get 21, 25, 29 and so on (but not 1,5,9,13 or 17). This makes 17 the largest not possible.