2022 Wits Mathematics Competition
Final Round
Upper Primary

Instructions

This paper is 90 minutes long and consists of ten single answer questions (to be answered in the below table) and two proofs (to be answered on the pages they're written on).

If needed, additional sheets of blank paper may be used to finish your solutions.

Calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed.

Questions 1 – 3 are each worth 4 marks.
Questions 4 – 7 are each worth 5 marks.
Questions 8 – 10 are each worth 6 marks.
Questions 11 – 12 are each worth 10 marks.
The total number of marks available is 70.

"It requires a very unusual mind to undertake the analysis of the obvious." - Alfred North Whitehead

Question	Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

A. 4 point questions

1. Palesa wanted to multiply a number by 101 but she forgot the 0 and multiplied the number by 11 instead. The resulting number was 99. What should her answer have been?
   Solution: 909. The initial number was 9, so the answer should have been 909.

2. What is the smallest ten-digit number that has exactly two digits that are the same and all other digits are different? Note: Numbers cannot start with zero!

   Solution: 1002345678.

3. In a soccer match between Kaizer Chiefs and Orlando Pirates, the final score is 3 : 2. How many possible scores are there at half-time?
   Solution: 12. Chiefs could have a score of any of the four numbers 0, 1, 2 or 3 and

   Pirates could have any of the scores 0, 1 or 2 with no score by one eliminating scores by the other.

B. 5 mark questions

4. Which number must replace the question mark if the total of the numbers in each row is the same?

1	2	3	4	5	6	7	8	9	10	199
11	12	13	14	15	16	17	18	19	20	?

Solution 99. Each of the first ten numbers in the top row is 10 less than it's counterpart. To balance this the ? must be 100 less than it's counterpart.

5. Today is a Monday. Thabo starts to read a book with 290 pages today. On Mondays he reads 25 pages and on every other day he reads 4 pages. On which day of the week does he finish reading the book?

Solution: A Saturday. Observe that Thabo reads a total of 49 pages a week. After six weeks (ending in a Sunday) he'd have read 296 pages if the book was long enough. The Saturday before he'd be on 292 pages and the Friday before on 288. Therefore he finishes on the sixth Saturday.

6. A box measures $2,8m \times 1,5m \times 2m$. What is the maximum number of smaller boxes measuring $0,3m \times 0,5m \times 0,7m$ that can fit into the bigger box?

Solution 48. You can fit four 0.7m sides on the 2.8m side of the larger box, four 0.5m sides on the 2m side of the larger box and five 0.3m sides on the 1.5m side of the larger box.

7. There are 60 learners in a class. Always two students share a desk. Every boy shares a desk with a girl. Exactly half the girls share a desk with a boy. How many boys are in the class?

Solution 20. There are as many boys as 'half the girls' (because they share desks). Boys are therefore a third of the class.

C. 6 mark questions

8. A series of 10 books were published at 2-year intervals. The sum of the publication years is 20000. When was the first book published?

   Solution: 1991 The average year of publication was 2000. This would be the time midway between the publication of the fifth and sixth books. So the books were published in: 1991, 1993, 1995, 1997, 1999, 2001, 2003, 2005, 2007, and 2009.

9. If we add the digits of 2022 we get $2 + 0 + 2 + 2 = 6$. How many years between 2000 and 9999 (including 2022) have digit sum 6?

   Solution: 35 The easiest way to solve this is to list them systematically. While it can also be solved by the so called 'stars and bars' method that requires knowing more mathematics than can reasonably be expected from junior secondary school students. However it's very much recommended that interested participants read up on it.

   It's worth mentioning that the 'systematic' way below is to always place the largest digit you can (haven't before first). This gives a list in reverse order.

   The possible answers are 6000, 5100, 5010, 5001, 4200, 4110, 4101,4020, 4011, 4002, 3300, 3210, 3201, 3120, 3111, 3102, 3030, 3021, 3012, 3003, 2400, 2310, 2301, 2220, 2211, 2202, 2130, 2121, 2112, 2103, 2040, 2031, 2022, 2013 and 2004

10. A cube is painted on the outside and then divided into unit cubes ($1 \times 1 \times 1$ cubes). The total number of painted faces equals the total number of unpainted faces. What was the side length of the cube before it was taken apart?

    Solution: $n = 2$. Notice that there are $6n^2$ outside faces and a total of $6n^3$ faces. Therefore we have $6n^2 = 3n^3$ which solves to $n = 2$.

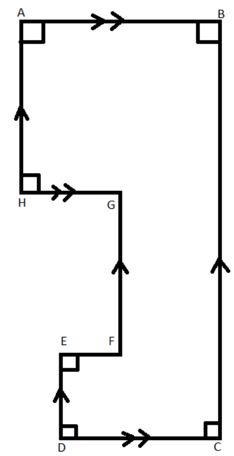
D. Proof questions, 10 marks each

11. Eight teams participate in a soccer tournament. Each team will play every other team in the tournament once in the group stage. After the group stage, the quarter finals will be played (1st vs $8th$, $2nd$ vs $7th$, etc), followed by the semi finals and a final. How many matches will be played in total? Show all your working.

Solution: 35 games. In the initial rounds each of the 8 teams play 7 games. This suggests 56 games but it counts the game X plays against Y and the game Y plays against X as different games (when they are in fact the same). We divide by 2 to accoutn for this and have 28 games in the initial seeding round.

After this we have 7 games which we can see by noticing that each player (except the overall winner) is kicked out in exactly one game. So the total number of games is $28 + 7 = 35$.

12. The lengths of the sides of the compound shape below are eight different whole numbers with the longest side being equal to $10cm$. What is the minimum area of the compound shape if $BC > AB > DC$ and $AH > ED$? Show all your working.



Solution: 23. We begin with some observations The first is that either BC must be the longest side and therefore have length 10 cm. We're given that it's longer than AB (which is clearly the longest horizontal line) and it's as long as the other vertical lines put together.

We can therefore observe that $CD + FG + AH = 10$ and that $BC = DE + FG + AH$ and $AB - GH = CD - EF$.

The next step is to realise that as FG is closer to BC than AH or CD that we want FG to be long. In fact the optimal will occur when $FG > AH > DE$ (or else they could be swopped).

A natural guess is therefore that $FG = 7$, $AH = 2$ and $DE = 1$. Putting $AB = 6$, $GH = 5$, $CD = 4$ and $EF = 3$. This gives an area of 23. This shows that FG must be exactly one away from BC because otherwise we'll certainly have a total area exceeding 23. Considering other permutations for the lengths of FG, AH and DE quickly eliminates them. So the final answer is in fact 23.