



2022 Wits Mathematics Competition
Qualifying Round
Middle Primary

Instructions

This exam consists of 20 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The mark allocation is as follows:

Questions 1-5 are each worth 3 points,
Questions 6-10 are each worth 4 points,
Questions 11-15 are each worth 5 points,
Questions 16-20 are each worth 6 points.
The total number of points available is 90.

The time limit on this exam is 75 minutes, calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed. If you are using the computer-friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense” — Alfred North Whitehead

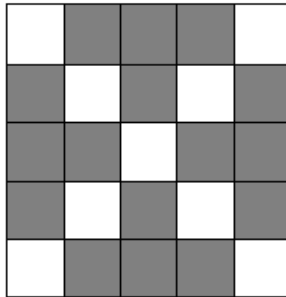
A. 3 point questions

1. What is the value of $43 - 12$?

- A) 21 B) 26 C) 31 D) 33 E) 35

Solution: C. $43 - 12 = 31$

2. In the picture below, how many more squares are shaded than not shaded?



- A) 6 B) 7 C) 8 D) 9 E) 10

Solution: B There are 16 shaded squares and 9 unshaded. $16 - 9 = 7$.

3. Malwande is competing in his school's cross country race and he has just overtaken the runner that was in 5th place. If Malwande then overtakes another 2 runners and gets overtaken by 4 runners, what is Malwande's final position?

- A) 3rd B) 4th C) 5th D) 6th E) 7th

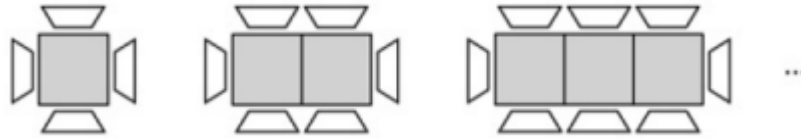
Solution: E. By taking over the runner in 5th place Malwande moves into 5th. He then takes over another 2 runners to get to the third place. He is then overtaken by 4 runners to wind up in seventh.

4. Tristan leaves home at 6:47 in the morning. It takes him 55 minutes to travel to school. He arrives 8 minutes before school starts. What time is school scheduled to start?

- A) 6:55 B) 7:42 C) 7:47 D) 7:50 E) 7:55

Solution: D. After the 55 minutes he takes to get to school it's 7 : 42. After another 8 minutes it's 7 : 50

5. A sequence of tables and chairs is shown below. How many chairs will there be in the fourth image?



A) 8

B) 9

C) 10

D) 11

E) 12

Solution: C. Each image gets an extra two chairs. Therefore the fourth (next) image will have 10

B. 4 point questions

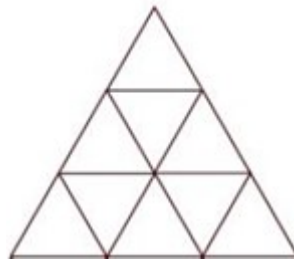
6. Alex opens his favourite mathematics puzzle book and notes that the product of the page numbers facing him is 132. Find the sum of these two page numbers. [Note: All pages are numbered consecutively.]

A) 12 B) 23 C) 45 D) 57 E) 71

Solution: B

The only two consecutive numbers we can multiply together to get 132 are 11 and 12. They sum to 23

7. How many triangles (of any size) are in the picture below?



A) 10 B) 11 C) 12 D) 13 E) 14

Solution: D. There are nine triangles with side length one, three with side length two and one with side length three. For a total of 13.

8. In the image below, which square can be removed to increase the total perimeter of the shape? Perimeter means the total distance around the shape.



A) Square A B) Square E C) Square F D) Square J E) Square K

Solution: C. Removing square F increases the perimeter by 2. Removing squares E or L would not change the perimeter and removing squares A or J would decrease the perimeter by 2.

9. Which fraction below is equal to $\frac{1}{3}$?

A) $\frac{2}{8}$

B) $\frac{3}{9}$

C) $\frac{5}{12}$

D) $\frac{6}{30}$

E) $\frac{12}{30}$

Solution: B. This can be shown by canceling a factor of three in both the numerator and denominator.

10. The Olympic committee printed numbered bibs for each of the athletes competing in the ski jump event, starting from the number 1. If 55 digits were printed overall, how many athletes competed?

A) 5

B) 17

C) 32

D) 45

E) 55

Solution: C

The first nine athletes take one digit per bib. Thereafter athletes will take two per bib (we won't get to 100 which is where they're require three). After printing 9 digits for the first 9 athletes we will have 46 digits left enough for the next 23 athletes. This gives a total of 32 athletes.

C. 5 point questions

11. Lungi and Kagiso are cousins, aged 10 and 34 respectively. How old will Lungi be when his age is exactly half of Kagiso's age?

A) 10 B) 22 C) 24 D) 34 E) 48

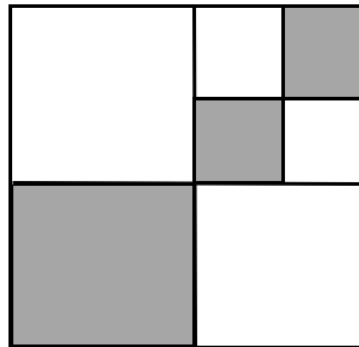
Solution: C. The difference will remain constant at 24 years and Lungi will be half Kagiso's age when his age is equal to the difference between their ages.

12. A box containing four Wits maths textbooks weighs 10kg, the same box with six Wits maths textbooks weighs 13kg. How much, in kg, does an empty box weigh, if all books have the same weight?

A) 1.5 B) 2 C) 3 D) 4 E) 6

Solution: D. Taking differences it's easy to see that two books weigh three kilograms. Therefore removing four books from the first box would decrease it's weight by six kilograms. Making an empty Box weigh four kilos.

13. The large square below has an area of 16cm^2 . It has been split into various smaller squares. Calculate the area, in cm^2 , of the shaded region.



A) 4 B) 6 C) 8 D) 12 E) 16

Solution: B. The bottom left square has area 4 (a quarter of the total). The two squares in the top right each have area 1 (a quarter of the larger square that they are in).

14. The first 56 odd numbers, starting from the number '1', are written. How many times does the digit '7' appear?

- A) 16 B) 18 C) 21 D) 26 E) 30

Solution: A. First notice that the 56th odd number is 111. We can then see that the 7 appears as a unit eleven times and as a tens five times. For a total of 16 times.

15. Along a highway, there are electric poles on one side of the highway and telegraph poles on the other side. The electric poles are separated by 15m and the telegraph poles are separated by 40m. If an electric and telegraph pole are both present at the point where the highway begins, after what distance, in m, will they be in front of each other again?

- A) 40 B) 60 C) 80 D) 120 E) 180

Solution: D. This can be done by computing the least common multiple of 15 and 40. As some younger students may not be familiar with this concept an alternate approach is list the electric pole distances (15,30,45,60,75,90,105,120,135) and the telephone poles are at (40, 80,120,160) and to observe that 120 is the first number to appear in both lists.

D. 6 point questions

16. A water tank is $\frac{7}{9}$ full. When 30 litres is released the tank is $\frac{5}{9}$ full. What is the full capacity of the tank (in litres)?

A) 30 B) 35 C) 135 D) 270 E) 900

Solution: C. The two ninths that were released equated to 30 litres. Hence a ninth is 15 litres and the full capacity is 105 litres.

17. A palindrome reads the same forwards and backwards. The number 202202 is a 6-digit palindrome. What is the difference of the palindrome directly before 202202 and the palindrome directly after 202202?

A) 1 B) 2 C) 2200 D) 200002 E) 202202

Solution: C. The palindrome immediately preceding 202202 is 201102 and the palindrome immediately after it is 203302. The difference between these is 2200. To see that these are the immediately preceding and succeeding palindromes observe that the first three digits of any six-digit palindrome force the next three and 202 lies next to and between 201 and 203

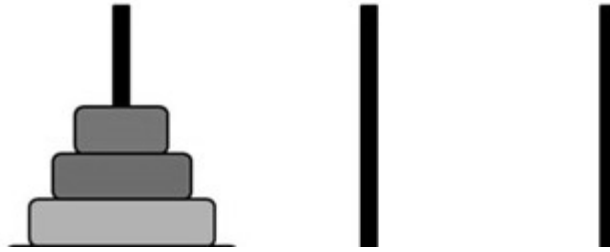
18. Tumelo makes a large cube from 27 small white cubes. She paints all the faces of the large cube (shaded in the picture below). She then removes a small cube from four corners, as shown. While the paint is still wet, she stamps each of the new faces onto a piece of paper. How many of the following stamps can Tumelo make?



A) 1 B) 2 C) 3 D) 4 E) 5

Solution: D. All but the last one can be made. To see that the last one can't be made observe that no side has only 5 remaining squares.

19. A tower of rings (called the Tower of Hanoi) is shown below. What is the minimum number of moves required to reform the tower (in the exact order of biggest ring at the bottom and smallest ring at the top) on either the second or third pole? You can only move one ring at a time, and may only place a smaller ring on top of a larger ring.



- A) 4 B) 5 C) 7 D) 8 E) 10

Solution: C. The only seven move sequence is as follows.

- Move the smallest block to the third tower
- Move the middle block to the second tower
- Move the smallest block to the second tower
- Move the largest block to the third tower
- Move the smallest block to the first tower
- Move the middle block to the third tower
- Move the smallest block to the third tower

To see that a six-move sequence does not exist observe that you need three moves before you can move the largest block (moving it on the fourth move is the earliest possible) and will need three after.

20. A stand sells movie popcorn in only 2 sizes. Their prices are R4 and R7 per serving. What is the greatest popcorn sales value, in Rands, that is NOT POSSIBLE?
- A) 15 B) 17 C) 23 D) 43 E) None

Solution: B. To see if an Rn purchase is possible we'll consider the remainder when n is divided by four. First notice that all multiples of 4 can be done, using only the 4. Now if we buy a single serving of size seven and some of size four, we see that 7,11,15,... can be done (but R3 cannot). Similarly if we buy two R7 servings We can get the numbers 14,18,22 and so on (but not 2, 6 or 10). Finally if we buy three servings of size 7 we can get 21, 25, 29 and so on (but not 1,5,9,13 or 17). This makes 17 the largest not possible.