2022 Wits Mathematics Competition
Final Round
Middle Primary

Instructions

This paper is 90 minutes long and consists of ten single answer questions (to be answered in the below table) and two proofs (to be answered on the pages they're written on). If needed, additional sheets of blank paper may be used to finish your solutions.

Calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed.

Questions 1 – 3 are each worth 4 marks.
Questions 4 – 7 are each worth 5 marks.
Questions 8 – 10 are each worth 6 marks.
Questions 11 – 12 are each worth 10 marks.
The total number of marks available is 70.

"It requires a very unusual mind to undertake the analysis of the obvious." - Alfred North Whitehead

Question	Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

A. 4 point questions

1. Aiden got the 11th highest mark out of a class of 47 students in their maths test. What was his rank from the bottom of the class? Solution: 37^{th}. There are 10 students above

   Aiden and therefore 36 below him. He is therefore ranked 37^{th} from the bottom.

2. A printer can print a book in 5 minutes. How long does it take for 100 of these printers to print 100 books?

   Solution: Five minutes. The 100 printers can each print one bookin the same five minute period.

3. Palesa wanted to multiply a number by 101 but she forgot the 0 and multiplied the number by 11 instead. The resulting number was 99. What should her answer have been?
   Solution: 909. The initial number was 9, so the answer should have been 909.

B. 5 mark questions

4. What is the value of $\frac{1}{2+\frac{1}{2+\frac{1}{2}}}$?

Solution: $\frac{5}{12}$

$$\begin{aligned}\frac{1}{2+\frac{1}{2+\frac{1}{2}}} &= \frac{1}{2+\frac{1}{\frac{5}{2}}} \\ &= \frac{1}{2+\frac{2}{5}} \\ &= \frac{1}{\frac{12}{5}} \\ &= \frac{5}{12}\end{aligned}$$

5. What is the sum of the largest 1-digit, 2-digit and 3-digit integer numbers?
Solution: 1107. The numbers are 9, 99 and 999.

6. The grid below contains a magic square. That means that each of the rows, columns and diagonals in the grid sum to the same total. Each of the digits in the grid are different (i.e. 1 to 9). The locations of digits 7 and 8 are given. What is the value of x?

		8
	7	x

Solution 6. The unique completed grid is shown below.

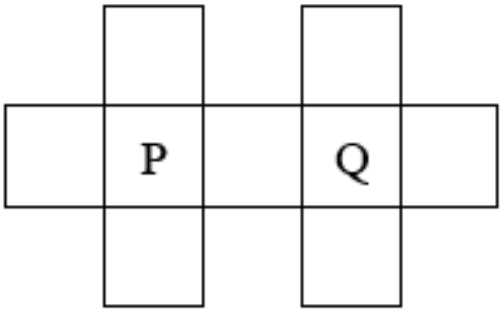
4	3	8
9	5	1
2	7	6

7. A box measures $2,8m \times 1,5m \times 2m$. What is the maximum number of smaller boxes measuring $0,3m \times 0,5m \times 0,7m$ that can fit into the bigger box?

   Solution 80. You can fit four 0.7m sides on the 2.8m side of the larger box, four 0.5m sides on the 2m side of the larger box and five 0.3m sides on the 1.5m side of the larger box.

C. 6 mark questions

8. Each digit from 1 to 9 is placed, one per box, in the diagram shown. The five numbers in the middle row add to 18. The first column of three adds to 16, and the last column adds to 22. What is the value of P + Q?



Solution: 11. The digits 1 to 9 add up to 45. Adding the five horizontal and the two verticle count every digit except that P and Q are counted twice. This sum is $18 + 16 + 22 = 56$ so P and Q sum to $56 - 45 = 11$

9. If $A * B$ means $\frac{A+B}{2}$, then what is the value of $(8 * 10) * 3$?

   Solution: 6

$$\begin{aligned}(8 * 10) * 3 &= (\frac{10+8}{2}) * 3 \\ &= 9 * 3 \\ &= \frac{(+3}{2} \\ &= 6\end{aligned}$$

10. A truck drives 60km from Johannesburg to Pretoria in 2 hours. The truck then makes the return trip from Pretoria to Johannesburg at an average speed of 90km/h. What was the average speed over the course of the entire trip?
    Solution: 45 km/h
    The return trip takes an extra 40 minutes or $\frac{2}{3}$ hour. Making the total trip $2\frac{2}{3} = \frac{8}{3}$ hours for 120km. This is $\frac{120}{\frac{8}{3}} = 45$ km/h

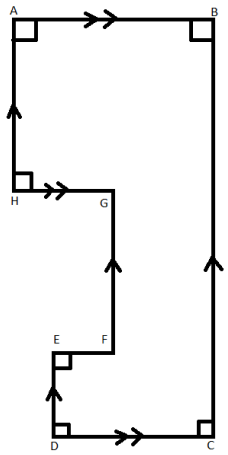
D. Proof questions, 10 marks each

11. Eight teams participate in a soccer tournament. Each team will play every other team in the tournament once in the group stage. After the group stage, the quarter finals will be played (1st vs 8th, 2nd vs 7th, etc), followed by the semi finals and a final. How many matches will be played in total?

    Solution: 35 games. In the initial rounds each of the 8 teams play 7 games. This suggests 56 games but it counts the game X plays against Y and the game Y plays against X as different games (when they are in fact the same). We divide by 2 to accoutn for this and have 28 games in the initial seeding round.

    After this we have 7 games which we can see by noticing that each player (except the overall winner) is kicked out in exactly one game. So the total number of games is $28 + 7 = 35$.

12. The lengths of the sides of the compound shape below are eight different whole numbers with the longest side being equal to $10cm$. What is the minimum area of the compound shape if $BC > AB > DC$ and $AH > ED$? Show all your working.



Solution: 23. We begin with some observations The first is that either BC must be the longest side and therefore have length 10 cm. We're given that it's longer than AB (which is clearly the longest horizontal line) and it's as long as the other vertical lines put together.

We can therefore observe that $CD + FG + AH = 10$ and that $BC = DE + FG + AH$ and $AB - GH = CD - EF$.

The next step is to realise that as FG is closer to BC than AH or CD that we want FG to be long. In fact the optimal will occur when $FG > AH > DE$ (or else they could be swopped).

A natural guess is therefore that $FG = 7$, $AH = 2$ and $DE = 1$. Putting $AB = 6$, $GH = 5$, $CD = 4$ and $EF = 3$. This gives an area of 23. This shows that FG must be exactly one away from BC because otherwise we'll certainly have a total area exceeding 23. Considering other permutations for the lengths of FG, AH and DE quickly eliminates them. So the final answer is in fact 23.