SHARP

2022 Wits Mathematics Competition
Qualifying Round
Junior Secondary

Instructions

This exam consists of 20 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The mark allocation is as follows:

Questions 1-5 are each worth 3 points,
Questions 6-10 are each worth 4 points,
Questions 11-15 are each worth 5 points,
Questions 16-20 are each worth 6 points.
The total number of points available is 90.

The time limit on this exam is 75 minutes, calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed. If you are using the computer-friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profoundity, he is talking nonsense" — Alfred North Whitehead

A. 3 point questions

1 A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1 + 2 + 4 + 7 + 14$. Which of the following is a perfect number?

   (A) 10   (B) 13   (C) 6   (D) 8   (E) 9

2 The smallest number in the set $\{0, -17, 4, 3, -2\}$ is

   (A) 4   (B) -17 (C) -2 (D) 0   (E) 3

3 What is the minimum number of digits that must be removed from the number 12323314, so that the resulting number is the same when read from either left to right or right to left?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

4 Tumelo makes a large cube from 27 small white cubes. She paints all the faces of the large cube (shaded in the picture below). She then removes a small cube from four corners, as shown. While the paint is still wet, she stamps each of the new faces onto a piece of paper. How many of the following stamps can Tumelo make?
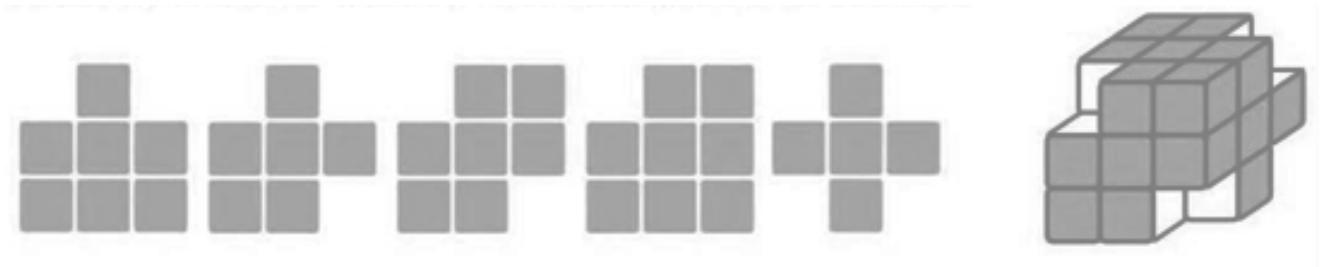
   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

5 The weight limit for an elevator is 1500 kilograms. The average weight of the people in the elevator is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the elevator?

   (A) 14   (B) 17   (C) 16   (D) 20   (E) 13

B. 4 point questions

6 Callan has 10 blue marbles, 6 green marbles and 5 red marbles in a bag. He draws one marble at a time and puts it aside. How many times should he draw to ensure that he has at least one marble of each colour?

   (A) 4   (B) 12   (C) 15   (D) 16   (E) 17

7 Compute $1000 - 999 + 998 - 997 + \cdots + 4 - 3 + 2 - 1$.

   (A) 999   (B) 500   (C) 250   (D) 2   (E) 1

8 Find x where $2^x - 2^{(x-3)} = 896$.

   (A) 7   (B) 8   (C) 9   (D) 10   (E) 11

9 On each of the sides of triangle ABC is constructed a semi-circle on the outside of the triangle with diameters AB, BC and CA respectively. If the three semi-circles have areas 8π, $\frac{25\pi}{2}$ and 18π, what is the perimeter of triangle ABC?

   (A) 18   (B) 20   (C) 24   (D) 30   (E) $\frac{77}{2}$

10 Suppose that x^* means $\frac{1}{x}$, the reciprocal of x. For example, $5^* = \frac{1}{5}$. How many of the following statements are true?

   (i) $2^* + 4^* = 6^*$ (ii) $3^* \times 5^* = 15^*$ (iii) $7^* - 3^* = 4^*$ (iv) $12^* \div 3^* = 4^*$

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

C. 5 point questions

11 When the product $5^3 \times 7^{52}$ is expanded, the units digit is

(A) 5   (B) 3   (C) 9   (D) 7   (E) 0

12 A number is *Beprisque* if it is the only natural number (positive whole number) between a prime number and a perfect square (e.g. 8 and 10 are Beprisque but 12 is not). The number of two-digit Beprisque numbers (including 10) is

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

13 How many 3 digit numbers (whole numbers between 100 and 999) are multiples of 6, 10 **and** 15?

(A) 18   (B) 20   (C) 30   (D) 33   (E) 60

14 $ABCD$ is a rectangle. Point E lies on AB such that angle $DEC = 90^o$. $DC = \sqrt{10}$ cm and $DE = 3$cm. Find the area of $ABCD$.

(A) $2\sqrt{10}$ (B) $\frac{3\sqrt{3}}{2}$ (C) 3   (D) 6   (E) 12

15 There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?

(A) 18   (B) 28   (C) 30   (D) 31   (E) 33

D. 6 point questions

16 How many zeroes will there be at the end of $1 \times 2 \times 3 \times 4 \times \cdots \times 39 \times 40$ when multiplied out (when all the natural numbers from 1 to 40 are multiplied together)?

(A) 6   (B) 7   (C) 8   (D) 9   (E) 11

17 In a triangle UWS the points I and T are placed on side WS so that $WU = WT$ and $SU = SI$. Determine $W\hat{U}S$ if $I\hat{U}T = 47^o$
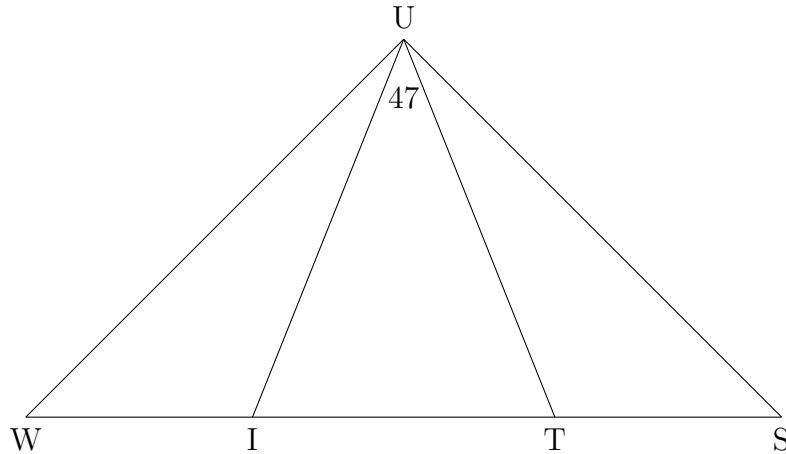
(A) 82   (B) 84   (C) 86   (D) 88   (E) 90

18 A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(A) $9 : 2$ (B) $9 : 4$ (C) $6 : 1$ (D) $3 : 1$ (E) $5 : 2$

19 The sum of the digits of the integer equal to

$$777\ 777\ 777\ 777\ 777^2 - 222\ 222\ 222\ 222\ 223^2$$

is

(A) 148   (B) 84   (C) 74   (D) 69   (E) 79

20 In trapezoid $ABCD$, AD is parallel to BC. Also, BD is perpendicular to DC. The point F is chosen on line BD so that AF is perpendicular to BD. AF is extended to meet BC at point E. If $AB = 41$, $AD = 50$ and $BF = 9$, what is the area of quadrilateral $FECD$?

(A) 900   (B) 960   (C) 1300   (D) 1523.5   (E) 1560