2022 Wits Mathematics Competition
Final Round
Junior Secondary

Instructions

This paper is 90 minutes long and consists of ten single answer questions (to be answered in the below table) and two proofs (to be answered on the pages they're written on).

   If needed, additional sheets of blank paper may be used to finish your solutions.

   Calculators may NOT be used. A ruler and compass may be used but all other geometric aids are NOT allowed. A translation aid (such as a dictionary) from English to another language is allowed.

Questions 1 – 3 are each worth 4 marks.
Questions 4 – 7 are each worth 5 marks.
Questions 8 – 10 are each worth 6 marks.
Questions 11 – 12 are each worth 10 marks.
The total number of marks available is 70.

   "It requires a very unusual mind to undertake the analysis of the obvious." - Alfred North Whitehead

Question	Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

A. 4 point questions

1. Which number must replace the question mark if the total of the numbers in each row is the same?

1	2	3	4	5	6	7	8	9	10	199
11	12	13	14	15	16	17	18	19	20	?

2. Today is a Monday. Thabo starts to read a book with 290 pages today. On Mondays he reads 25 pages and on every other day he reads 4 pages. On which day of the week does he finish reading the book?

3. In a football match between Kaizer Chiefs and Orlando Pirates, the final score is 3 : 2. How many possible scorelines are there at half-time?
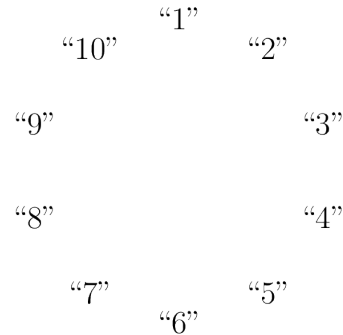
B. 5 mark questions

4. How many whole numbers between 2000 and 3000 have digits that sum to 6?

5. ABCD and DEFG are squares. $CE = 17cm$ and the area of the shaded region ABCGFE is $161cm^2$. What is the perimeter of the shaded region in cm?
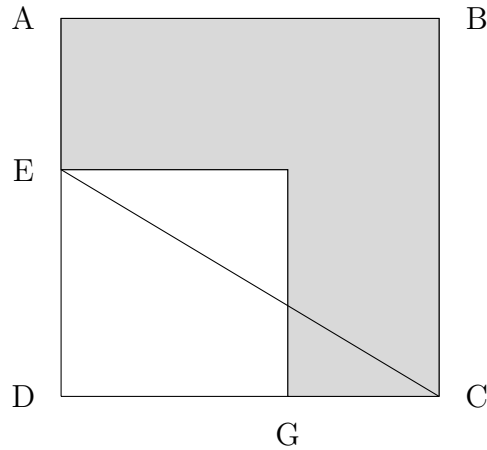
6. How many positive integers are factors of 20! (Note: ! denotes multiplying by one less number each time until you get to 1. For example, 4! is $4 \times 3 \times 2 \times 1$)?

7. Adele and Bongi play a game. The game ends when either Adele or Bongi win two consecutive rounds. The probability that Adele wins a round is 0.3, while the probability that Bongi wins a round is 0.6. The probability that a round is drawn is 0.1. What is the probability that neither is the winner after at most three games?

C. 6 mark questions

8. Ten people form a circle. Each picks a random number and tells it to the two neighbours adjacent to them in the circle. Then each person computes and announces the average of the numbers of their two neighbours. The figure shows the average announced by each person (not the original number the person picked!)

$$
\begin{array}{ccc}
 & \text{``1''} & \\
\text{``10''} & & \text{``2''} \\
\text{``9''} & & \text{``3''} \\
\text{``8''} & & \text{``4''} \\
\text{``7''} & & \text{``5''} \\
 & \text{``6''} &
\end{array}
$$

What was the number picked by the person who announced the average of 6?

9. The stick of length 2.5m is split into two pieces. The break point is equally likely to be anywhere along the stick's length. After this the lengths of the two sticks are measured and the lengths (in meters) are rounded to the nearest integer. What is the probability that the two rounded integers add up to 3?

10. Let P_1 be a regular polygon with r sides and P_2 be a regular polygon with s sides, with $r \geq s \geq 3$. Each interior angle of P_1 is $\frac{59}{58}$ as large as each interior angle of P_2. What is the largest possible value of s?

D. Proof questions, 10 marks each

11. Each of the letters in the equation $FORTY + TEN + TEN = SIXTY$ represents a different digit from 0 through 9. The digits are concatenated so for example if $A = 1$, $N = 3$ and $D = 8$, then $AND = 138$. Further in our example letters always mean the same thing. So the Y in $FORTY$ is the same digit as the Y in $SIXTY$. Find which letter corresponds to which number so that the equation is true. Show that your solution is the only possible solution. Show all work

12. What is the sum of the integers k for which the expression below is also an integer?

$$\frac{(k^2 + 2k - 6)^2}{k + 1}$$