

WMC 2019 Junior Secondary Final Round Solutions

Section A

1. **25**

This can be done by the Sieve of Eratosthenes.

2. **420**

In the numerator $1 \times 2 \times \cdots \times 19$ cancels out and we have $20 \times 21 = 420$ remaining.

3. **40m**

The width of the rectangle is $91 \div 13 = 7$. So the perimeter of the rectangle is $2 \times (7 + 13) = 2 \times 20 = 40$.

4. **80 minutes**

In an hour the eldest sister can paint a third of the wall, the middle sister can paint a quarter of the wall and the youngest sister can paint a sixth of the wall. So together they can paint

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4 + 3 + 2}{12} = \frac{9}{12} = \frac{3}{4}$$

of the wall in an hour meaning that they can paint a quarter of the wall in $60 \div 3 = 20$ minutes. So to paint the whole wall they would need $20 \times 4 = 80$ minutes.

5. **1944**

These will be the cubes in on the outer layer except the corners and the sides. On one of the four faces there are $(20 - 2)^2 = 18^2 = 324$ such cubes, so in total we will have $324 \times 6 = 1944$ such cubes.

6. **27**

The trick here is to write $999 \times 777 = 1000 \times 777 - 777 = 777000 - 777 = 776223$. So the sum of the digits is $7 + 7 + 6 + 2 + 2 + 3 = 27$.

7. **5416**

We partition these numbers by the first time a 5 or a 6 appears. There are 2000 numbers that begin with a 5 or a 6. There are 2000 numbers that begin with a 5 or a 6. There are $7 \times 2000 = 1400$ numbers that have a 5 or 6 appear first in the hundreds digit. There are $7 \times 8 \times 2 \times 10 = 1120$ that have a 5 or a 6 first occur as the tens digit. Finally there are $7 \times 8 \times 8 \times 2 = 896$ have a 5 or a 6 first appear as the units digit. So in total there are $2000 + 1400 + 1120 + 896 = 5416$ such numbers.

8. **671**

Let x be the smallest number. Then the other two numbers are $x + 2$ and $x + 4$. So $x + (x + 2) + (x + 4) = 2019 \implies 3x + 6 = 2019 \implies 3x = 2013 \implies x = 2013 \div 3 = 671$.

9. **13**

Let d be the length of the main diagonal. The diagonal of the face with sides 3 and 4 is $\sqrt{3^2 + 4^2} = 5$. Then $d^2 = 5^2 + 12^2 \implies d = 13$.

10. **1458**

Observe that the sum of the children's ages must be a multiple of 9 since the number formed by putting the ages together is a multiple of 81 and so is a multiple of 9. So the possible age sums are 9, 18, 27 and 36. We cannot go any higher since each age is a single digit so it is at most 9. It remains then to deal with each of these cases.

Section B

11. The archer will have to at some point have an average accuracy of 75 percent. Suppose that initially the archer has hit the target a out of b times where $\frac{a}{b} < \frac{3}{4}$. Suppose that the archer plays n more games with

$$\frac{a+n}{b+n} < \frac{3}{4} \implies 4a+n < 3b \implies 4a+n \leq 3b-1$$

and

$$\frac{a+n+1}{b+n+1} > \frac{3}{4} \implies 4a+n+1 > 3b$$

which cannot happen.

12. In 20 minutes, the eldest sister can paint $\frac{1}{9}$ of the wall, the middle sister can paint $\frac{1}{12}$ of the wall and the three sisters together can paint $\frac{1}{4}$ of the wall. So the youngest sister can paint $\frac{1}{4} - \frac{1}{9} - \frac{1}{12} = \frac{1}{18}$ of the wall so she will take $18 \times 20 = 360$ minutes to paint the wall by herself.