2020 Wits Mathematics Competition
Final Round
Grade 10-12
Name:
School:

Instructions

This paper is 90 minutes long and consists of ten single answer questions (to be answered in the below table) and two proofs (to be answered on the pages they're written on). If needed, additional sheets of blank paper may be used to finish your solutions. Geometric equipment and language dictionaries are allowed but calculators and other computing devices are not.

"The really unusual day would be one where nothing unusual happens.". — Persi Diaconis

Question	Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

A. Single Answer Questions

2 Marks

1. The number of girls in a classroom is more than 40 % and less than 50% of the number of learners in the class. What is the minimum number of learners in this classroom?

2. How many real solutions does $\frac{x^3}{\sqrt{4-x^2}} + x^2 - 4 = 0$ have?

3. The acute triangle ABC has $\hat{C} = 45°$. E is a point on BC where $AE \perp BC$ and D is a point on AC where $BD \perp AC$. Determine the length of DE if $CE = 4$cm and $EB = 3$cm.

3 Marks

4. What is the area of a circle that passes through the point $(4; \sqrt{5})$ and externally touches the circle $x^2 + y^2 = 9$ (centered at the origin with radius 3) at the point $(2; \sqrt{5})$?

5. We have a deck of $n > 1$ cards and a collection of symbols such that the following conditions are satisfied:

    1. Three distinct symbols are drawn on each card.
    2. Any two cards have exactly one symbol in common.
    3. Any two symbols appear together on exactly one card.

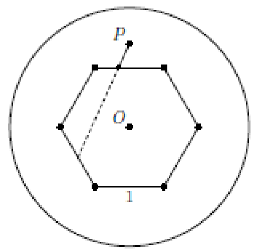
    What is the value of n?

6. Find all pairs of integers $(x; y)$ satisfying $xy + 3x - 5y = 17$.

7. How many different ways can 369 be written as the sum of a number of consecutive natural numbers?

4 Marks

8. A regular hexagon of side length 1 sits inside a circle with area 3π . The hexagon and the circle are both centred at O. A point in the circle can "see" an edge of the hexagon if it is possible to draw a straight line from that point to the edge, without crossing any of the other edges. For example, the point P can see the edge directly beneath it, but it is blocked from seeing the lower left edge. Given a randomly chosen point in the circle, what is the probability that it can see exactly two edges?



9. Find all prime numbers, p, such that $p^3 + p^2 + 11p + 2$ is prime?

10. What is the remainder when $3^3 + 5^3 + 7^3 + ... + 1999^3$ is divided by 999000?

B. Proof Questions

11. Prove that $15x^2 - 7y^2 = 9$ has no integer solutions.

12. A semicircle has diameter AB. C and D are points on the semicircle such that $AC = CD$, and the tangent through C intersects BD produced in E. Line segment AE intersects the semicircle at point G. Prove that $CG < GD$.