2020 Wits Mathematics Competition
Final Round
Grade 8-9
Name:
School:

Instructions

This paper is 90 minutes long and consists of ten single answer questions (to be answered in the below table) and two proofs (to be answered on the pages they're written on). If needed, additional sheets of blank paper may be used to finish your solutions. Geometric equipment and language dictionaries are allowed but calculators and other computing devices are not.

"The really unusual day would be one where nothing unusual happens.". — Persi Diaconis

Question	Answer
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

A. Single Answer Questions

2 Marks

1. What is the smallest natural number W so that $204 \times W$ is a perfect square?
2. Find the smallest value of n such that $2^{400} \cdot 5^n$ has 404 digits when expanded.
3. For how many integers x is the expression $\sqrt{1400 - \sqrt{x}}$ an integer?

3 Marks

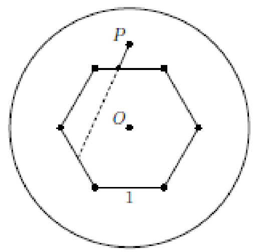
4. A train leaves K for L at 09 : 30 while another train leaves L for K at 10 : 00. The first train arrives at L 40 minutes after the trains pass each other. The second train arrives K 1 hour and 40 minutes after the trains pass each other. Each train travels at a constant speed. At what time do the trains pass each other?
5. The number of girls in a classroom is more than 40 % and less than 50% of the number of learners in the class. What is the minimum number of learners in this classroom?
6. An autobiographical number is a number whose first digit is the number of zeros in the number, whose second digit is the number of ones, and so on. For example 1210 is autobiographical because it has 1 zero, 2 ones, 1 two and 0 threes. It just so happens that there is exactly one 10 digit autobiographical number. What is this number?
7. How many different ways can 369 be written as the sum of a number of consecutive natural numbers?

4 Marks

8. We have a deck of $n > 1$ cards and a collection of symbols such that the following conditions are satisfied:
   1. Three distinct symbols are drawn on each card.
   2. Any two cards have exactly one symbol in common.
   3. Any two symbols appear together on exactly one card.

   What is the value of n?

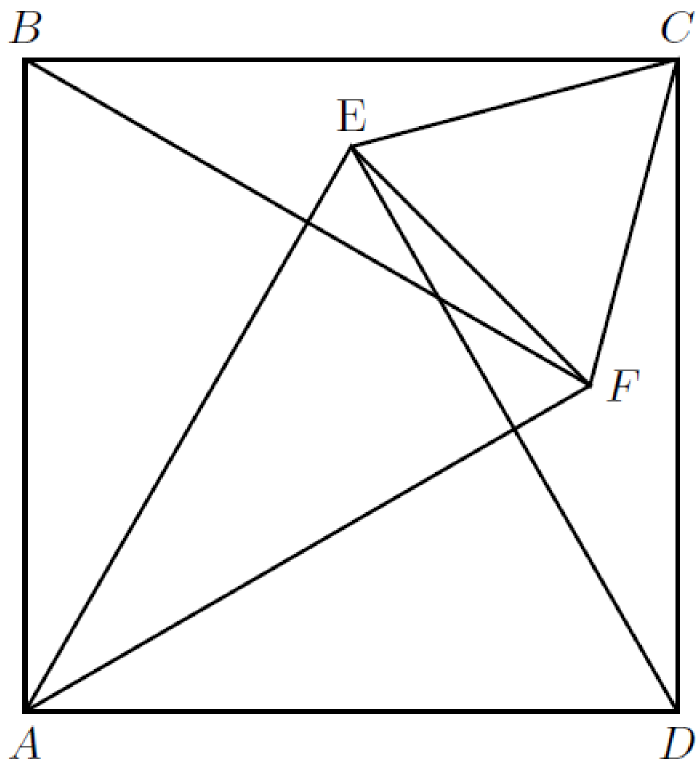
9. A regular hexagon of side length 1 sits inside a circle with area 3π . The hexagon and the circle are both centred at O. A point in the circle can "see" an edge of the hexagon if it is possible to draw a straight line from that point to the edge, without crossing any of the other edges. For example, the point P can see the edge directly beneath it, but it is blocked from seeing the lower left edge. Given a randomly chosen point in the circle, what is the probability that it can see exactly two edges?



10. The acute triangle ABC has $\hat{C} = 45°$. E is a point on BC where $AE \perp BC$ and D is a point on AC where $BD \perp AC$. Determine the length of DE if $CE = 4$cm and $EB = 3$cm.

B. Proof Questions

11. In the diagram below, $ABCD$ is a square. Triangles AFB and AED are equilateral. Prove that triangle EFC is equilateral.



12. If x, y and z are integers with $x^3 + y^3 + z^3$ divisible by 18. Prove or disprove that xyz is divisible by 6.