# WMC 2020 Upper Primary Fianl Round Solutions

## Section A

1. **50**

   $(2+4+\cdots+98+100)-(1+3+\cdots+97+99) = (2-1)+(4-3)+\ldots(98-97)+(100-99) = 1+1+\cdots+1+1 = \frac{100}{2}\times 1 = 50$.

2. **90°**

   Consider the bottom right triangle. Its angles are $90^\circ$, $180^\circ-90^\circ-y^\circ = 90^\circ-y^\circ$ and $180^\circ-90^\circ-x^\circ = 90^\circ-x^\circ$. And so $90^\circ+(90^\circ-y^\circ)+(90^\circ-x^\circ) = 180^\circ \implies 270^\circ-(x^\circ+y^\circ) = 180^\circ \implies x^\circ+y^\circ = 270^\circ-180^\circ = 90^\circ$.

3. **10**

   Let $s$ be Sihle's number.
   Then $2s+10 = 3s \implies 10 = 3s-2s \implies s = 10$.

4. **24**

   Since one seat is already taken by Oti, there are four seats left to place her four friends. For her first friend, there are 4 choices for her seat. For the second friend, there will be 3 options for her seat since 1 of the 4 seats is already taken. For the third friend, there are 2 seat choices and for the last friend there is only one choice. So in total there are $4\times 3\times 2\times 1 = 24$ arrangements.

5. $\mathbf{\frac{28}{15}}$

   Let $b, g$ and $t$ be the number of boys, girls and teachers respectively. Then $\frac{b}{g} = \frac{4}{5}$ and $\frac{g}{t} = \frac{7}{3}$. Now multiplying these two equations gives $\frac{b}{g}\times\frac{g}{t} = \frac{4}{5}\times\frac{7}{3} \implies \frac{b}{t} = \frac{28}{15}$.

6. **8**

   Let the numbers be $a, b, c$ and $d$ listed in ascending order. Then the median of the four numbers is $\frac{b+c}{2} = 6$ and so $b+c = 12$. Now the mean of the four numbers is $\frac{a+b+c+d}{4} = 5 \implies a+b+c+d = 20$. So $a+d = 20-12 = 8$.

7. **22**

   Let the width of the rectangle be $x$ and its length be $y$. The perimeter of the rectangle is $2x+2y$. The perimeter of the square is $2\times(2+5+x+3+1+y) = 2x+2y+22$. So the difference between the perimeters is 22.

8. **7**

   Let the number of cards be $n$ and the number of symbols be $k$. Then by (3), we have that $\binom{k}{2} = \frac{k(k-1)}{2} = 3n \implies n = \frac{k(k-1)}{6}$. Also each symbol must appear exactly $m = \frac{3n}{k}$ times. And so

$\binom{n}{2} = \frac{n(n-1)}{2} = k \times \binom{m}{2} = \frac{km(m-1)}{2} = \frac{3n(\frac{3n}{k}-1)}{2} \implies n(\frac{9}{k}-1) = 2$. So $(\frac{k(k-1)}{6})(\frac{9}{k}-1) = 2 \implies k = 3$ or $k = 7$. Now $k = 3$ gives $n = 1$ which cannot be, and $k = 7$ gives $n = 7$. A unique construction exists.

9. $\mathbf{\frac{1}{6}}$

   Suppose that the three equal parts of the rectangle are equal to $x$ and that the other side of the rectangle is equal to $y$. Then to total area of the rectangle is $(x + x + x) \times y = 3xy$ and the area of the shaded triangle is $\frac{xy}{2}$. So the proportion of the shaded region is $\frac{\frac{xy}{2}}{3xy} = \frac{1}{6}$.

10. **51**

    $204 = 2^2 \times 3 \times 17$. Now any perfect square is the product of distinct primes with even powers. So we need to at least multiply 204 by $3 \times 17 = 51$ in order to get a perfect square.

## Section B

11. The only 10 digit autobiographical number is 6210001000.
    We do trial and error on the first digit. We start when it is as large as possible since the larger cases are simpler. If the first digit is 9 then all the other digits must be 0 and so the number cannot be autobiographical. If the first digit is 8 then the $9th$ digit must be 1 and the rest of the digits must be 0 which is not an autobiographical number. When the first digit is 7 the $8th$ digit must be at least 1 and 7 of the remaining 8 digits must be 0. This leaves that the only possibility is 71000001000 which is not an autobiographical number. Now if we try the first digit to be 6 we get 6210001000 which is valid.

12. We have that $\angle FBC = \angle EDC = 90° - 60° = 30°$. Since $DE = AD = CD$, triangle $CED$ is isosceles. Then $\angle DCE = \frac{180° - \angle CDE}{2} = 75° \implies \angle ECB = 15°$. Similarly, we have $\angle FCD = 15°$ and so $\angle ECF = 60°$. Also since $\triangle DEC \equiv BFC$ we have that $CE = FC$ and so $\triangle EFC$ is isosceles with one angle equal to $60° \implies \triangle EFC$ is equilateral.