WMC 2020 Middle Primary Final Roound Solutions

Section A

1. **25**

   $1034 \div 42 = 24$ remainder 26 and so we well need 25 pages for the list

2. **3**

   Three boys can plant one plant per minute, meaning that the same number of boys can plant 60 plants in 60 minutes.

3. **180**

   Suppose they meet after t hours, Stefan has walked $4t$ km and Thandi has walked $3t$ km. Then their distances must add up to 21 $\implies$ $4t + 3t = 21$ $\implies$ $7t = 21$ $\implies$ $t = 3$. So they meet after $3 \times 60 = 180$ minutes.

4. **12**

   d For the first digit, there are four options. Then for the second digit there are three options since we have already taken one card. So we can make $4 \times 3 = 12$ different cards.
   Alternatively, we can list all the numbers $12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43$.

5. **25**

   Suppose that each of the squares have an area of 8. Then the top left has a shaded area of $\frac{1}{4} \times 8 = 2$. The top left has a shaded area of $\frac{1}{4} \times 8 = 2$. The top right has a shaded area of $\frac{1}{8} \times 8 = 1$. The bottom left has a shaded area of $\frac{3}{48} \times 8 = 3$. The bottom right has a shaded area of $\frac{1}{4} \times 8 = 2$. So the percentage of the shaded area is $(2 + 1 + 3 + 2) \div (8 \times 4) \times 100 = 25$.

6. **83**

   Let the digits be a and b. Then $a + b = 11$ and $10b + a = 10a + b - 45$ $\implies$ $a - b = 5$. Adding these two equations gives $2a = 16$ $\implies$ $a = 8$ and so $b = 3$. So the two-digit number is 83.

7. **Sunday**

   From the $3rd$ of April to the $8th$ of September, there are $27 + 31 + 30 + 31 + 31 + 8 = 127$ days. Now $127 \div 7 = 18$ remainder 1 so the $8th$ of September falls one day of the week after the day on which that the $3rd$ of April falls. So it falls on a Sunday.

8. $\mathbf{\frac{1}{4}}$

   The area of the bottom triangle is a quarter of the total area of the square so it is equal to $\frac{1}{4}$. The area of the two side triangles is equal to a quarter of the area of the square each. So the sum of areas of the three triangles is $\frac{1}{4} \times 3 = \frac{3}{4}$ and so the area of the shaded region is $1 - \frac{3}{4} = \frac{1}{4}$.

9. **9**

   The number in the last column is the sum of the squares of the first two columns in the same row. So the missing number is $\sqrt{170 - 7^2} = 9$

10. **R8**

    The cost of two chips and two juices is $R28$ so the cost of one chips and one juice is $R14$. The total cost of one of each item is $R22$ and so the cost of one chocolate is $22 - 14 = R8$.

Section B

11. The only 10 digit autobiographical number is 6210001000.
    We do trial and error on the first digit. We start when it is as large as possible since the larger cases are simpler. If the first digit is 9 then all the other digits must be 0 and so the number cannot be autobiographical. If the first digit is 8 then the $9th$ digit must be 1 and the rest of the digits must be 0 which is not an autobiographical number. When the first digit is 7 the $8th$ digit must be at least 1 and 7 of the remaining 8 digits must be 0. This leaves that the only possibility is 71000001000 which is not an autobiographical number. Now if we try the first digit to be 6 we get 6210001000 which is valid.

12. We count how many numbers have 2 or 3 as factors then subtract them from the total number of two digit numbers. The numbers divisible by 2 are $10, 12, 14, \ldots, 98$ which counts to $98 \div 2 - 4 = 37$ numbers. The numbers divisible by 3 are $12, 15, \ldots, 99$ which counts to $99 \div 3 - 3 = 30$ numbers. Now some numbers are divisible by both 2 and 3, these are multiplies of 6 which are $12, 18, \ldots 96$ and there are $96 \div 6 - 1 = 15$ numbers. So the number of numbers that are divisible by 2 or 3 is $37 + 30 - 15 = 52$ numbers. Therefore the number of numbers that are divisible by neither is $90 - 52 = 48$.
    Alternatively one may list all these 48 numbers.