WMC 2020 Junior Secondary Qualifying Round Solutions

Section A

1. **D** The simplest way to do this is by testing the options one by one. Another approach is to compute that 506 leaves a remainder of 2 when divided by 12 (we say that 506 is congruent to 2 modulo 12) and that adding 10 lowers the remainder by 2. This means that the options for A are 1 and 7 and 1 isn't a listed option.

2. **E** Notice that POQ is isosceles so $\angle PQO = 30^o$, which makes $x = 60$ (right angles) and $y = \angle POQ = 180 - 2 \times 30 = 120$.

3. **A** The key insight is that every fold doubles the number of holes and $2^5 = 32$.

4. **E** This may be seen by plugging in any value less than -1.

5. **C** $1024 = 2^{10} \leq 2020 < 2^{11} = 2048$

6. **D** There are only 7 squares less than 50 (1,4,9,16,25,36 and 49). This leaves only 21 possible combinations (some of which are eliminated by being over 50). Systematically writing this down gives the six primes 5, 13, 17, 29, 37 and 41.

7. **D** There are $2 \times \binom{5}{2} = 20$ triangles that have the top vertex as the top vertex of the big triangle since each of those triangles is uniquely defining by choosing two points on the bottom line or the slanting line. There is one more small triangle in the bottom left. So in total there are 21 triangles.

8. **A** The leftmost digit can only be 5 or 6 since if it greater than that we will have no more digits to fill to the right since they must be increasing. If the first digit is 5 we can choose 3 digits from $\{6, 7, 8, 9\}$ and arrange them in ascending order. We can choose these digits in 4 ways. If the first digit is 6 the only option is 6789. So there are 5 such numbers.

9. **B** Let us call the rectangles sides b and l. As the perimeter is 56 we can set $l = 14 + d$ and $b = 14 - d$ for some d. This gives an area of $196 - d^2 = 192$, which gives $d = 2$. The side lengths are therefore 12 and 16 and the difference between them is 4.

10. **D** Call the initial price x. The Manager wanted the price to be $0.75x$ but it's currently $1.25x$ so the decrease must be $0.5x$ and the percentage is $\frac{0.5}{1.25} \times 100 = 40$.

11. **E** To see this split the perimeter into four groups (left, right, top and bottom, by which par of a square they are on). It is easy to see that the top and bottom group each have a total length of $5cm$ while the left and right groups are of length $2cm$. This gives a total of $16cm$.

12. **A** Observe that the area is a right angled triangle with the corner where the two squares meet being the right angle (it's a combination of two forty-five degree angles). This makes the shades area a right angled triangle with base and height $5\sqrt{2}$ and $7\sqrt{2}$. Hence a total area of $35cm^2$.

13. **A** There are 12 ways to choose the first person and 11 ways to chose the second. However this counts every pair twice because choosing A and then B is the same as choosing B and then A. So $\frac{12\times 11}{2} = 66$.

14. **A** We split this into three cases by the person who missed.

$$\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\times+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}=\frac{6}{2\times 3\times 4}=\frac{1}{4}$$

15. **B** The length of the two shortest rods is at least 1. For minimality we set them both to equal 1. Now the third shortest is $1+1$ (by the triangle inequality). Similarly the fourth shortest rod is at least $2+1=3$. This forms a pattern of the Fibonacci numbers and so the longest rod is at least the $10th$ Fibonacci number which is 55.