

WMC 2019 Upper Primary Final Round Solutions

Section A

1. **21**

This can be done by long division.

2. **25**

This can be done by the Sieve of Eratosthenes.

3. **420**

In the numerator $1 \times 2 \times \cdots \times 19$ cancels out and we have $20 \times 21 = 420$ remaining.

4. **40m**

The width of the rectangle is $91 \div 13 = 7$. So the perimeter of the rectangle is $2 \times (7 + 13) = 2 \times 20 = 40$.

5. **6**

The pages that were removed are 25, 26, 27, 28, 29, 30 so 6 pages were removed.

6. **R270**

A 20 percent increase would mean the book costs $\frac{120}{100} \times 300 = R360$. Now if this price decreases by 75 percent it would cost $\frac{75}{100} \times 360 = R270$.

7. **6**

These are the numbers in the form $\frac{x}{81}$ where $x = 9 \times k$ where k and 81 have no factors in common. So the possible values for k are 1, 2, 4, 5, 7 and 8. So there are 6 such numbers.

8. **$\frac{7}{18}$**

If the two dice roll a and b , there are 36 possible outcomes since for each die there 6 possible outcomes. Then we can observe a prime if the sum is 2, 3, 5, 7 or 11. Any other prime is too large.

Now we can observe a score of 2 if we get (1, 1).

We can observe a score of 3 if we get (1, 2) or (2, 1).

We can observe a score of 5 if we get (1, 4), (2, 3), (3, 2) or (4, 1).

We can observe a score of 7 if we get (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) or (6, 1).

We can observe a score of 11 if we get (5, 6) or (6, 5). In total we have 14 pairs that give a prime score, so the probability is $\frac{14}{36} = \frac{7}{18}$.

9. **1296**

These will be the cubes in on the outer layer except the corners and the sides. On one of the four faces there are $(20 - 18)^2 = 18^2 = 324$ such cubes, so in total we will have $324 \times 4 = 1296$ such cubes.

10. **671**

Let x be the smallest number. Then the other two numbers are $x + 2$ and $x + 4$. So $x + (x + 2) + (x + 4) = 2019 \implies 3x + 6 = 2019 \implies 3x = 2013 \implies x = 2013 \div 3 = 671$.

Section B

11. In an hour the eldest sister can paint a third of the wall, the middle sister can paint a quarter of the wall and the youngest sister can paint a sixth of the wall. So together they can paint

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4 + 3 + 2}{12} = \frac{9}{12} = \frac{3}{4}$$

of the wall in an hour meaning that they can paint a quarter of the wall in $60 \div 3 = 20$ minutes. So to paint the whole wall they would need $20 \times 4 = 80$ minutes = 1 hour and 20 minutes.

12. Write the sum as $(2019^{2019} + 1^{2019}) + (2018^{2019} + 2^{2019}) + \dots + (1011^{2019} + 1009^{2019}) + 1010^{2019}$. Now the factorisation of $a^n + b^n$ when n is odd includes the factor $a + b$ so all of these sums are divisible by 2020 and so they are divisible by 10 meaning their last digit is 0, so the last digit of the sum is also 0.