WMC 2019 Senior Secondary Qualifying Round Solutions

1. **B** From 5:20 pm to 6:00 pm there are 40 minutes and from 6:00 pm to 6:05 pm there are 5 minutes. Altogether there are $40 + 5 = 45$ minutes.

2. **D** Consider a square with side length x with area x^2. After increasing its side length by 25% its side length is $1,25x$ and so the area is $1,5625x^2$.

3. **B** $0,2019 - 0,02019 = \frac{2019}{100000} \times (10 - 1) = 2019 \times 9 \div 100000 = 0,18171$

4. **D** There are 9 learners in front of Thabo and there are 11 learners behind him. We must also count Thabo himself. In total there are $11 + 9 + 1 = 21$ learners.

5. **D** Approximate 7982413 to 8000000 and approximate 0,000246 to 0,00025. Then $8000000 \times 0,00025 = 8 \times 25 \times 10 = 2000$.

6. **C** It is possible to get the solution here by careful counting. However an approach that will work more generally (on larger grids) is to choose two of the vertical line (which can be done in $\binom{4}{2} = 6$ ways) and two horizontal lines (again there are 6 ways to do this). To get $6 \times 6 = 36$ rectangles.

7. **D** One way to do this is to write out all the terms until the $83rd$ term.
However a shorter way is to keep on adding the the numbers up to n until you go beyond 83 and then you stop.

8. **D** $736 \times 32 + 726 \times 68 = 726 \times 100 = 72600$

9. **B** This can be done by factorizing the polynomial to $(x - 1)(x - 2)(x - 3)$, so the roots are 1,2 and 3 which sum to 6. Another approach is to realize that $(x-a)(x-b)(x-c)$ multiplies out to $x^3-(a+b+c)x^2+(ab+ac+bc)x-abc$, and as we're looking for $a+b+c$ we can simply read it off the coefficient of x^2 and flip it's sign.

10. **E** For each handshake there are 14 choices for the first person and 13 choices for the second person. So there are 14×13 choices. However since we can choose the same pair of people in two different ways, the number of handshakes is $\frac{14\times13}{2} = 91$.

11. **B**

$$\begin{aligned} S &= 9 + 99 + 999 + \cdots + 99\ldots99 \\ &= (10^1 - 1) + (10^2 - 1) + \cdots + (10^{2019} - 1) \\ &= 10^1 + 10^2 + \cdots + 10^{2019} - 2019 \\ &= 1111\ldots110 - 2019 \\ &= 1111\ldots11109091 \end{aligned}$$

12. **A** Rationalising the denominators gives the sum

$$\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\ldots\sqrt{25}-\sqrt{24}=\sqrt{25}-\sqrt{1}=4$$

13. **D** Begin by setting angle $ABF = y$ and $FDE = z$. $A = 180° - 2y$ and $E = 180° - 2z$ and ACE is a triangle this gives $y + z = 130°$ as AFE is a straight line this makes $x = 50°$.

14. **A** It's possible to get 60 (by getting everything). It's clearly not possible to get 58 or 59. 57, 56 and 55 can be achieved by getting all but one question correct. Finally 54 can be achieved by getting two questions from section A wrong and everything else correct.

15. **A** The trick is to realize that powers of go in cycles. Powers of 2 end in 2, 4, 8, 6 and then 2 again. A little calculation reveals that 2^{2019} will end in an 8. Similarly powers of 3, end in 3, 9, 7 and 1 with 3^{2019} ending in a 7. Lastly powers of 5 always end in 5 (a cycle of length one). Adding these together gives a final last digit of 0.