2019 Wits Mathematics Competition
Final Round
Grades 8 and 9
Time: 90 Minutes

Instructions

This exam consists of 12 questions. The first 10 are single answer and are worth 3 marks each. The last 2 are proof questions which require full solutions. They are out of 10 marks each.

"A mathematician is a device for turning coffee into theorems." — Paul Erdos

Full Name: _____
School: _____
Division: **Junior Secondary**
Grade: _____
E-mail: _____

Answer Section A below

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

A. Single Answer Question

1. How many prime numbers are there between 1 and 100?
2. Compute $\frac{1\times2\times3\times...\times21}{1\times2\times3\times...\times19}$
3. A rectangular swimming pool has area $91m^2$ and length $13m$. Find it's perimeter.
4. Three sisters decide to paint a wall. The eldest sister could have painted it herself in 3 hours, the middle sister would have taken 4 hours and the youngest would have taken 6. How long will it take them to paint the wall working together?
5. A $20 \times 20 \times 20$ cube is constructed by sticking 8000 unit cubes together. The cube is subsequently dropped in paint and then broken up into the original 8000 cubes. How many cubes have exactly 1 painted side?
6. Evaluate the sum of the digits of the product 999×777.
7. How many four digit numbers contain either a 5 or a 6 (or both)?
8. 2019 can be written as a sum of 3 consecutive odd numbers. Find the smallest of these.
9. Find the length of the main diagonal in a rectangular prism, with side lengths 3,4 and 12.
10. Grandma Mathematics is 81 years old and has 4 grandchildren. If the sum of the grandchildrens' ages is multiplied by the grandmother's age it gives a 4-digit number whose digits are the ages of her grandchildren. Find the 4 digit number.

B. Proof Questions

11. A beginner archer calculates that he has hit the target less than 75 percent of the time (over his entire career of shooting up to that point). After some more practice sessions he gets better and eventually recalculates his (again lifetime) hit rate. He is pleased to notice that he's now hit the target more than 75 percent of the time. Is he guaranteed to have hit it at exactly 75 percent of the time at some point between the two times he calculated it? Give a proof that the average must be exactly 75 percent at some point or provide a counter-example.

12. Three sisters decide to paint a wall. The eldest sister could have painted it herself in 3 hours, the middle sister would have taken 4 hours and working together the three sisters can paint it in 80 minutes. How long would it take the youngest sister to paint the wall alone?