2019 Wits Mathematics Competition
Final Round
Grades 10,11 and 12
Time: 90 Minutes

Instructions

This exam consists of 12 questions. The first 10 are single answer and are worth 3 marks each. The last 2 are proof questions which require full solutions. They are out of 10 marks each.

"A mathematician is a device for turning coffee into theorems." — Paul Erdos

Full Name:

School:

Division:　　　　　　　　　　　　　　　　　**Senior Secondary**

Grade:

E-mail:

Answer Section A below

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A. Single Answer Question

1. Evaluate the sum of the digits of the product 999×777.

2. A rectangular swimming pool has area $91m^2$ and length $13m$. Find it's perimeter.

3. How many prime numbers are there between 1 and 100?

4. How many four digit numbers contain either a 5 or a 6 (or both)?

5. Find the smallest positive integer that is divisible by 6 different primes.

6. Identical regular pentagons are placed together side by side in the manner shown. The diagram shows 2 pentagons. How many are required to make a full ring?
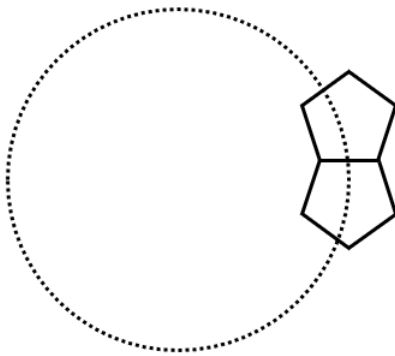
7. At a pet daycare some puppies and kittens are playing. Initially 20 percent of the animals are puppies. However three new puppies join the group and three kittens are taken away. Which changes things to 30 percent puppies. How many kittens where originally in the group?

8. Find a 6-digit number N such that the numbers $2N$, $3N$, $4N$, $5N$ and $6N$ consist of the same digits as N but written in a different order.

9. Grandma Mathematics is 81 years old and has 4 grandchildren. If the sum of the grandchildrens' ages is multiplied by the grandmother's age it gives a 4-digit number whose digits are the ages of her grandchildren. Find the 4 digit number.

10. Seven marked points P_1,P_2,P_3,P_4,P_5,P_6 and P_7 lie on a line in that order. A marked point P is placed on the line and the quantity S is calculated as the sum of the lengths of the seven line segments $\overline{PP_1}$, $\overline{PP_2}$,$\overline{PP_3}$,$\overline{PP_4}$,$\overline{PP_5},\overline{PP_6}$ and$\overline{PP_7}$. Describe the set of choices for P such that S is minimal.

B. Proof Questions

11. 2019 people are about to board a plane with 2019 seats numbered 1 through 2019, They have tickets numbered by the integers 1 through 2019 inclusive. The first person enters the plane and instead of sitting in seat 1 chooses a seat at random. The rest of the passengers are better behaved, they will sit in their assigned seat if possible. If they can't sit in their assigned seat they will sit in a random empty seat (each empty seat is equally likely to be picked). What is the probability that passenger 2019 sits in seat 2019?

12. (a) Sixteen players enter a chess contest. They each have a different skill level and for purposes of this question a better player will always beat a weaker player. The contest takes the format of a 4 round knockout tournament. Where the players are set against each other randomly. Find the probability that the third best player is eliminated by the best.

(b) How does this differ if 256 players play in an 8 round knockout tournament?

(c) If 2^n players compete in an n round tournament?