Wits Mathematics Competition

Undergraduate **9 May 2018**

Time Limit: 75 Minutes

Full Name: ______________________

E-mail: ______________________

Seat Number: ______________________

University: ______________________

Registed Degree: ______________________

Year: ______________________

Instructions

This exam consists of 3 sections. Section A contains 10 multiple choice questions for 3 marks each. Section B consists of 10 single answer questions for 5 marks each. Section C consists of two questions which require full workings, each for 10 marks. You should answer Sections A and B on this page and section C on the sheets the questions are printed on.

Scores

Section	Mark	Perfect
A		30
B		50
C		20
Total		100

Problems worthy of attack prove their worth by fighting back - Piet Hein.

Section A [30 Marks]

Multiple Choice Questions					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Section B [50 Marks]

Single Answer Questions	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

A. Multiple Choice

1. Find the sum of the first 2018 terms of the sequence $1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \ldots$.

   A. 5048

   B. 5012

   C. 5043

   D. 5029

   E. 5061

2. x and y are distinct positive integers less than 20. If $y = x^3$ determine the value of y^2.

   A. 16

   B. 25

   C. 36

   D. 49

   E. 64

3. Which number must be removed from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ so that the average of the remaining ten is $6, 1$?

   A. 1

   B. 2

   C. 3

   D. 5

   E. 8

4. Seven children are born on the same day of the year in seven consecutive years. The sum of the ages of the youngest three is 42. How old is the oldest?

   A. 16

   B. 17

   C. 18

   D. 19

   E. 20

5. If 40% of P is 10% of Q, what percentage of P is Q?
   A. 18%
   B. 22%
   C. 25%
   D. 26%
   E. 30%

6. The base 7 representation of a positive integer has 2 digits. Reversing those digits gives the base 5 representation the same number. How many such numbers have a base 10 representation less than 2018?
   A. 1
   B. 5
   C. 72
   D. 118
   E. 144

7. What are the last 2 digits of 2018^{2018} ?
   A. 24
   B. 32
   C. 64
   D. 72
   E. 74

8. If the ratio of the surface areas of two cubes is k, then the corresponding ratio of the volumes is :
   A. $k^{1/3}$
   B. $k^{3/2}$
   C. k^2
   D. k^3
   E. None of the above

9. If $(2^a)(2^b) = 256$, determine the mean of a and b?
   A. 3
   B. $3, 5$
   C. 4
   D. $4, 5$
   E. 5

10. An equilateral triangle is circumscribed about a circle of radius 9. What is the area of the triangle?

    A. $27\sqrt{(3)}$

    B. $81\sqrt{(\frac{3}{4})}$

    C. $243\sqrt{(3)}$

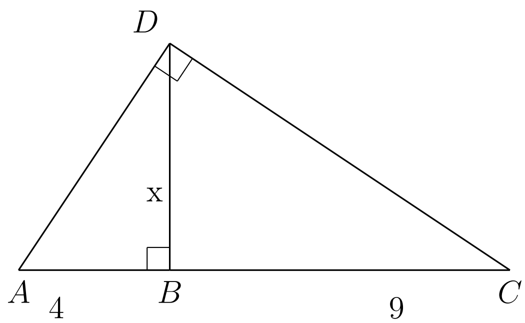
    D. $729\sqrt{(\frac{3}{4})}$

    E. None of the above

B. Single Answer

11. An octagonal swimming pool has sides which are, consecutively 10 m, 20 m, 30 m, 40 m, 50 m, 60 m, 70 m and 80 m. All the pool's angles are right angles. Find the top surface area of the pool, in square metres.

12. Find the value of x, if ADC and ABD are right angles.



13. Let a, b be arbitrary real numbers, and let the sequence x_n be defined by $x_0 = a$, $x_1 = b$, and $x_n = (x_{n-1} + x_{n-2})/2$ for $n \geq 2$. Find a formula for the limit of x_n in terms of a and b.

14. A fair coin is flipped 7 times. Find the probability of getting exactly 4 heads.

15. Find the number of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ that are subsets of neither $\{1, 2, 3, 4, 5\}$ nor $\{4, 5, 6, 7, 8\}$.

16. When $0,\overline{123}$ is written in rational form in lowest terms, the sum of the numerator and denominator is:

17. An orange is 80% water (by mass). If 75% of the water is evaporated, what percentage of the orange is now water?

18. Find the number of positive integers less than or equal to 2018 whose base-three representation contains no digit equal to 0.

19. Find in terms of $p > 1$ the value of the following fraction:

$$\frac{1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + ..}{1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + ..}$$

20. The sum of the first n terms of the sequence 1, (1+2), $(1+2+2^2)$,...,$(1+2+...+2^{k-1})$,...is of the form $2^{n+R} + Sn^2 + Tn + U$ for all $n > 0$. Find $R + S + T + U$.

C. Proof Questions

21. A 10×10 square grid of light bulbs is initially off. For every 3×3 connected sub-grid you have a switch which will toggle every light bulb within that grid. That is, it will change the bulbs state, either from on to off or off to on. You have a similar switch for every 5×5 sub-grid. Is it possible to get any arrangement of on/off bulbs by flipping these switches? Provide a proof.

22. Which members of the sequence 101,10101,1010101,101010101,... are prime? Provide a proof.